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A Fuzzy EPQ Model for Non-Instantaneous Deteriorating Items

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Abstract

The inventory system has been drawing more intrigue because this system deals with the decision that minimizes the total average cost or maximizes the total average profit. For any firm, the demand for any item depends upon population, selling price and frequency of advertisement, etc. In most of the models, it is assumed that deterioration of any item in inventory starts from the beginning of their production. But in reality, many goods maintain their good quality or original condition for some time. So, price discount is availed on defective items. Our target is to calculate the total optimal cost and the optimal inventory level for this inventory model in a crisp and fuzzy environment. Here holding cost is taken as constant and no-shortages are allowed. The cost parameters are considered as Triangular Fuzzy Numbers and to defuzzify the model, Signed Distance Method is applied. A numerical example of the optimal solution is given to clarify the model. The changes of different parameters effect on the optimal total cost are presented and sensitivity analysis is given.

Keywords: EPQ Inventory, Non-Instantaneous Deterioration, Demand dependent Production, Defuzzification, Signed Distance Method

JEL Classification: C44, Y80, C61

Paper Classification: Research Paper

Introduction

In an EPQ inventory, it is important to control quality. Most of the models of the inventory control system are formulated with the assumption that all produced items are of good quality. But in reality, for any production company to produce all good quality products is impossible. On the other hand, due to the different phenomenons, there are so many goods which deteriorate after their lifetime. In such a situation, price discount is a common practice by the supplier that encourages the customer to purchase defective and deteriorated items other than regular purchase. So the effect of deterioration and defective items cannot be ignored in inventory models.

Most inventory models considered the request rate to be either stock needy or consistent or time-subordinate. It has been observed that decrease in the cost of the item for the most part positively affects demand of the item. It becomes a necessity to make a proper strategy to maintain the inventory economically. Ghare et. al. (1963) developed an inventory model for the exponentially decaying inventory system. These types of models were extended and improved by Misra (1975). The investigators generally have taken the demand as constant. In reality, demand always depends on selling price of an item, population of that area, deterioration, the frequency of advertisement of the product, etc. As time advanced, a few researchers created inventory models with deteriorating items, shortage items, demand patterns, cost patterns, items order cycles and their combinations. Bhunia et.al. (2014) derived a deterministic inventory model where deteriorated items demand depends upon the selling price of items and the frequency of advertisement. On the other hand, to reduce the cost, an intelligent businessman or a production company always produces products depending on demand.

Without any ambiguity, many inventory model based on different kinds of vulnerabilities are classically modelled using the approaches from the likelihood hypothesis. Some of the business fit such conditions, yet applying these models as they may be, for the most part, prompts incorrect choices. Here fuzzy inventory models fulfil that gap and can get more exact outcomes for inventory problems, rather than the conventional likelihood hypothesis by using fuzzy set theory. It was presented by Zadeh (1965), whose research work has been receiving considerable attention from investigators in production and inventory system. Bellmann et. al. (1970) proposed a scientific model of decision making in fuzzy condition. Later, Dubois et. al. (1978) defined some operations on fuzzy numbers. Zimmermann (1985) made an attempt to use the fuzzy sets in operation research. Syed et. al. (2007) investigated a fuzzy inventory model without shortages using signed distance method. Dutta et. al. (2012) contributed to fuzzy inventory model without shortage using trapezoidal fuzzy number. Maragatham et. al. (2014) researched a fuzzy inventory model for deteriorating items with price-dependent demand.

Motivation & Contribution of Study

In the proposed model, a fuzzy deterministic stock model for non-instantaneous deteriorating things with production proportional to demand is shown. Variable demand pattern depends on population, selling price and frequency of advertisement which is variable or constant according to any real-life situation. Here they are treated as constants. So, production company produces items according to demand. On the other hand, defection and deterioration occur for any production. In such a situation price discount is a common phenomenon. The inventory parameters are taken as the triangular fuzzy number. Signed distance method is used to defuzzify the model. The goal for finding the solution for minimizing the total cost has been derived. Such type of model has not yet been discussed in the inventory literature.

Definitions and Fuzzy Preliminaries

Definition 2.1: A fuzzy set $\tilde{A}$ is a universe of discourse X is defined as the following set of pairs $\tilde{A}=\{(x,\mu_{\tilde{A}}(x):x\in X\}$. Where $\mu_{\tilde{A}}(x)\to[0,1]$ is a mapping called the membership function of the set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\tilde{A}$. The larger $\mu_{\tilde{A}}(x)$ is stronger the grade of membership form in $\tilde{A}$.

Definition 2.2 : A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, $x_2 \in X$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)]$ when $0\leq\lambda\leq 1$.

Definition 2.3: A fuzzy set $\tilde{A}$ of the universe of discourse X is called normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x)=1$.

Definition 2.4: α- level set : The α-cut of $\tilde{A}$ is defined as a crisp set $A_\alpha=\{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X$ where $\alpha \in [0,1]$. A_α is a non-empty bounded closed interval contained in X and it can be denoted by $A_\alpha=[A_L(\alpha), A_R(\alpha)]$. Where $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval respectively.

Definition 2.5: A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. The following figure 1 shows a fuzzy number $\tilde{A}$.

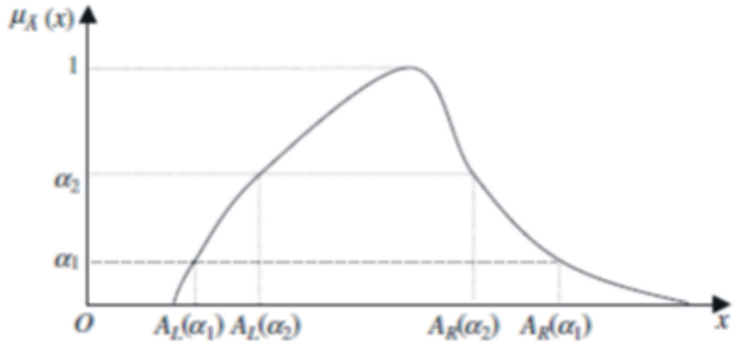

Fig. -1:- Fuzzy number $\tilde{A}$ With α-cuts.

Figure-1 shows a fuzzy number $\tilde{A}$ with α-cuts $A_{\alpha 1}=[A_L(\alpha_1), A_R(\alpha_1)]$, $A_{\alpha 2}=[A_L(\alpha_2), A_R(\alpha_2)]$. It is seen that if $\alpha_2 \geq \alpha_1$ then $A_L(\alpha_2) \geq A_L(\alpha_1)$ and $A_R(\alpha_2) \geq A_R(\alpha_1)$.

Definition 2.6: The function principle is used for the operation for Addition, Subtraction, Multiplication and Division of fuzzy numbers. Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then_

(i) **Addition:** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$, where a1, a_2, a_3; b_1, b_2, b_3 are any real numbers.

(ii) **Subtraction:** $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$, where a_1, a_2, a_3; b_1, b_2, b_3 are any real numbers.

(iii) **Multiplication**: $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$, where a_1, a_2, a_3; b_1, b_2, b_3 are all non-zero positive real numbers.

(iv) **Division:** $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$, where b_1, b_2, b_3 are all non-zero positive real numbers.

(v) **Scalar Multiplication:** For any real number K,

$K\tilde{A} = (Ka_1, Ka_2, Ka_3)$, Where $K \geq 0$,

$K\tilde{A} = (Ka_3, Ka_2, Ka_1)$ Where $K < 0$,

Definition 2.7: The α-cut of $\tilde{A}$ is defined by $A_\alpha=\{x:\mu_{\tilde{A}}(x) = \alpha, \alpha \geq 0\}$.

Definition 2.8: Among the various shapes of fuzzy number, triangular fuzzy number (TFN) is the most popular one. $\tilde{A}$ is represented by the triplet (a1, a2, a3) and is defined by its continuous membership function where $\mu_{\tilde{A}}(x):X \rightarrow [0,1]$ is given by

$$\mu_{\tilde{A}}(x) = f(x) = \begin{cases} 1 - \frac{a_2 - x}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } x = a_2 \\ 1 - \frac{x - a_1}{a_2 - a_1}, & \text{for} a_2 \le x \le a_3 \\ 0, & \text{for Otherwise} \end{cases}$$

Definition 2.9: The α-level set of the triangular number $\tilde{A}=(a_1, a_2, a_3)$ is :

$\tilde{A}= \{x: \mu_{\tilde{A}}(x) \ge \alpha\}=[A_L(\alpha), A_R(\alpha)]$.

Where$A_L(\alpha)=a_1+(a_2-a_1)\alpha, \alpha\in[0,1]$, And $A_R(\alpha)=a_3-(a_3-a_2)\alpha, \alpha\in[0,1]$.

We represent $\tilde{A}=(a_1, a_2, a_3) = \cup[A_L(\alpha), A_R(\alpha)]; 0\le\alpha\le1$.

Definition 2.10: Defuzzification of $\tilde{A}$ can be found by Signed Distance Method. If $\tilde{A}$ is a triangular fuzzy number then sign distance from $\tilde{A}$ to 0 is defined as:

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [\{A_L(\alpha), A_R(\alpha)\}, 0]\, d\alpha$$

Where, $\tilde{A}=[A_L(\alpha), A_R(\alpha)]$ and $\tilde{A}=[a_1+(a_2-a_1)\alpha, a_3-(a_3-a_2)\alpha]$, $\alpha\in[0,1]$ is α-cut off fuzzy set $\tilde{A}$, which is a close interval.

Notations and Assumptions

This inventory model is produced based on the accompanying assumptions and notations which are utilized in Crisp and Fuzzy Environment.

Notations :

- $I(t)$: The inventory level at any time t, $t \ge 0$.
- C_1: The fixed operating cost of the inventory.
- C_2: The advertisement cost per advertisement.
- lp : The production cost per unit per unit time.
- Tac: The total average cost per unit per cycle.
- (C_1): The Fuzzy fixed operating cost of the inventory.
- (C_2) : Fuzzy advertisement cost per advertisement.
- (Tac): Fuzzy total average cost per unit per cycle.
- t_1: The production time when the quality of products in stock reaches maximum L_m, $t_1 > 0$.
- t_2: The time duration where there is no production but deteriorating and end of t_2 the inventory level diminished gradually to zero, $t_2 > 0$.
- t_1+t_2 : The length of cycle time, $t_1+t_2 > 0$.

Assumptions :

- The rate of non-instantaneous decay whenever any time $t > 0$ is time proportional, $\theta(t) = \beta t$; where, $\beta(0 < \beta < 1)$ is the scale parameter.
- The demand rate $D(m,p,f)= mf/p$ is dependent on population (m), selling price (p) of an item and the frequency of advertisement (f), where m, p, f > 0.

- Production rate K(k,m,p,a)=kD(m,p,f)=k mf/p, where k is a positive constant.
- Holding cost is h, a constant.
- Lead time is zero or negligible.
- The discounted rate d per unit per unit time.
- The Defective items rate r per time for each cycle.
- The horizontal planning takes place at an infinite rate.
- There is no replenishment or repair of deteriorating and defective items in the given cycle.
- The lead time is considered zero.

Production Inventory Model in Crisp Environment is produced as follow

Let, the producer start to produce items at the start of each cycle when t = 0 to satisfy the arriving demands in the inventory system. At the end of time t_1, the production stopped where some produced items are defective. The inventory level is assumed to reach to its highest level L_m (>0) at end of t_1. During the time interval t_2, the inventory level diminishes owing to customer demand and deterioration and finally falls to zero at $t = t_1+t_2$. Figure – 2 delineates the inventory level of the proposed model.

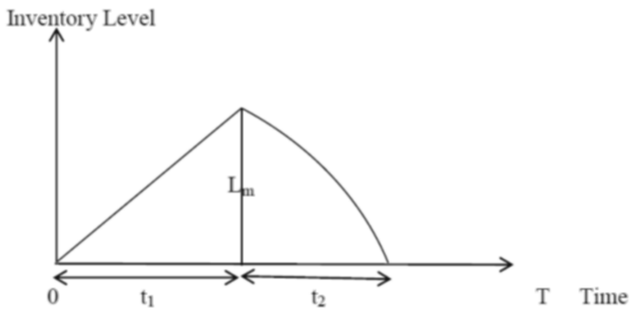

Figure – 2 : Design of Proposed Inventory System

The Inventory Level in $t_1(0 \leq \tau \leq t_1)$: The produced items during t_1 would be depleted due to the instant demand as well as defective items. Under above assumption, during the period t_1, the inventory status of the system is given by the following differential equation-

$$dI_1(t)/dt = kD(m,p,f) - D(m,p,f) - r, \quad \text{for } (0 \leq t \leq t_1) \tag{1}$$

From the initial Condition $I_1(0) = 0$ and $I_1(t_1) = L_m$ got from above equation (1), and

$$I_1(t) = \left(k\frac{mf}{p} - \frac{mf}{p} - r\right)t, \qquad \text{for } (0 \leq t \leq t_1) \tag{2}$$

$$L_m = \left(k\frac{mf}{p} - \frac{mf}{p} - r\right)t_1, \tag{3}$$

The Inventory Level in t_2 $(t_2 \leq t \leq t_1+t_2)$: In this time, the inventory declines due to customers' demand and deterioration. Hence, the status of the inventory level during t_2 is governed by the following Differential Equation,

$$dI_2(t)/dt + \beta I_2(t) = -D(m,p,f), \quad \text{for } (t_1 \leq t \leq t_1+t_2) \tag{4}$$

From the boundary condition I_2 (t_1+t_2) = 0 and dismissing the higher intensity of β and taking initial two terms of the exponential series, we get,

$$I_2(t) = [\frac{mf}{p}]\,[t_1 + t_2 - t + \beta\frac{t^3}{3} + \frac{\beta(t_1+t_2)^3}{6} - \beta(t_1 + t_2)\frac{t^2}{2}], \quad (5)$$

Now the costs functions are :

1. The Operating cost during the period [0, t_1+t_2] : C_1 (6)
2. The Production cost during the period [0, t_1] : lpk mf/p t_1=klmft_1, (7)
3. The Inventory Holding Cost during the period $[0, t_1 + t_2]: \int_0^{t_1} hI_1(t)dt + \int_{t_1}^{t_1+t_2} hI_2(t)dt$. Using equation (2) and (5), then integrating, becomes the Holding Cost,

$$h\left(k\frac{mf}{p} - \frac{mf}{p} - r\right)\frac{t_1^2}{2} + h\frac{mf}{p}\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1 + t_2)\frac{{t_1}^3}{6}\} \quad (8)$$

4. The Deteriorating Cost during the period $[t_1,\ t_1 + t_2]: lp\int_{t_1}^{t_1+t_2} \beta tI_2(t)dt$. Using equation (5) and integrating, the Deteriorating Cost becomes

$$\beta lmf[\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2] \quad (9)$$

5. The Advertisement cost during the period [0, t_1+t_2] : C_2 f (10)

6. The Price Discount during the period $[t_1, t_1 + t_2]: lpd\int_{t_1}^{t_1+t_2}\frac{mf}{p}dt$ From above, Price Discount is ldmft_2 (11)

Therefore the total average cost function per cycle : 1/(t_1+t_2) [Operating Cost + Production Cost + Inventory Holding Cost + Deteriorating Cost + Advertisement Cost + Price Discount].

Hence the average net cost function is

$$Tac\,(t_1, t_2) = \frac{1}{(t_1+t_2)}[C_1 + klmft_1 + h\left(k\frac{mf}{p} - \frac{mf}{p} - r\right)\frac{t_1^2}{2} + h\frac{mf}{p}\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1 + t_2)\frac{{t_1}^3}{6}\} + \beta lmf[\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2] + C_2f \pm ldmft_2], \quad (12)$$

Now, the necessary condition for the total average cost function of the system is minimized if equation (12) is satisfying

$$\partial Tac\,(t_1,\ t_2)/(\partial t_1) = 0, \quad (13)$$

And $\partial Tac\,(t_1,\ t_2)/(\partial t_2) = 0,$ (14)

The solution, which might be called feasible solution of the problem, of the conditions (13) and (14) give the optimal solutions of $t_1=t_1^*$ and $t_2=t_2^*$ which minimize Tac (t_1, t_2) = Tac (t_1, t_2)* provide they satisfy the sufficient conditions-

$$\frac{\partial^2 Tac\,(t_1,\ t_2)}{\partial t_1^2}\cdot\frac{\partial^2 Tac\,(t_1,\ t_2)}{\partial {t_2}^2} - \left(\frac{\partial^2 Tac\,(t_1,\ t_2)}{\partial t_1\,\partial t_2}\right)^2 > 0, \quad (15)$$

And $\frac{\partial^2 Tac\,(t_1,\ t_2)}{\partial t_1^2} > 0\ or,\ \frac{\partial^2 Tac\,(t_1,\ t_2)}{\partial {t_2}^2} > 0,$ (16)

However, it's difficult to solve the problem by inferring an explicit equation of the solutions from conditions (13) and (14). Therefore, the optimal service level of $t_1 = t_1^*$ and $t_2 = t_2^*$ is solved by using the software LINGO 17.0. Moreover, it is also verified that the sufficient conditions of the optimality of the solutions of t1=t1* and t2=t2* are satisfied (i.e. inequalities (15) and (16)) under certain conditions

The proposed Inventory Model in Fuzzy Environment is produced as follow

Presently the above model will be produced in fuzzy Environment. Due to uncertainly, it is difficult to characterize every one of the parameters definitely. It is assumed that, $\widetilde{C_1} = (C_1^1, C_1^2, C_1^3)$, $\tilde{h} = (h^1, h^2, h^3)$, $\tilde{p} = (p^1, p^2, p^3)$, $\widetilde{C_2} = (C_2^1, C_2^2, C_2^3)$,is Triangular Fuzzy Number in LR-form then the total average cost function of the system per unit time in fuzzy environment is given by-

$$\widetilde{Tac(t_1,\ t_2)} = \frac{1}{(t_1+t_2)}[\widetilde{C_1} + klmft_1 + \tilde{h}\left(k\frac{mf}{\tilde{p}} - \frac{mf}{\tilde{p}} - r\right)\frac{t_1^2}{2} + \tilde{h}\ \frac{mf}{\tilde{p}}\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{{t_1}^3}{6}\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + \widetilde{C_2}f \pm ldmft_2] \quad (17)$$

$$\text{Or, } \widetilde{Tac(t_1,\ t_2)} = \frac{1}{(t_1+t_2)}[(C_1^1, C_1^2, C_1^3) + klmft_1 + (h^1,h^2,h^3)\left(k\frac{mf}{(p^1,p^2,p^3)} - \frac{mf}{(p^1,p^2,p^3)} - r\right)\frac{t_1^2}{2} + (h^1,h^2,h^3)\ \frac{mf}{(p^1,p^2,p^3)}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{{t_1}^3}{6}\right\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + (C_2^1, C_2^2, C_2^3)f \pm ldmft_2] = (U, V, W)\ (\text{Say}) \quad (17)$$

Where,

$$U = \frac{1}{(t_1+t_2)}[C_1^1 + klmft_1 + h^1\left(k\frac{mf}{p^1} - \frac{mf}{p^1} - r\right)\frac{t_1^2}{2} + h^1\ \frac{mf}{p^1}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{{t_1}^3}{6}\right\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + C_2^1 f \pm ldmft_2];$$

$$V = \frac{1}{(t_1+t_2)}[C_1^2 + klmft_1 + h^2\left(k\frac{mf}{p^2} - \frac{mf}{p^2} - r\right)\frac{t_1^2}{2} + h^2\ \frac{mf}{p^2}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{{t_1}^3}{6}\right\} + \beta lmf\left\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\right\} + C_2^2 f \pm ldmft_2];$$

And,

$$W = \frac{1}{(t_1+t_2)}[C_1^3 + klmft_1 + h^3\left(k\frac{mf}{p^3} - \frac{mf}{p^3} - r\right)\frac{t_1^2}{2} + h^3\ \frac{mf}{p^3}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{{t_1}^3}{6}\right\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + C_2^3 f \pm ldmft_2];$$

The α – cuts, $A_L(\alpha)$ and $A_R(\alpha)$ of triangular fuzzy number $\widetilde{Tac(t_1,\ t_2)}$ are given by-

$$A_L(\alpha) = U + (V-U)\alpha = \frac{1}{(t_1+t_2)}[C_1^1 + klmft_1 + h^1\left(k\frac{mf}{p^1} - \frac{mf}{p^1} - r\right)\frac{t_1^2}{2} + h^1\ \frac{mf}{p^1}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{{t_1}^3}{6}\right\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + C_2^1 f + ldmft_2] + \frac{1}{(t_1+t_2)}[(C_1^2 - C_1^1) + h^2\left(k\frac{mf}{p^2} - \frac{mf}{p^2} - r\right)\frac{t_1^2}{2} - h^1\left(k\frac{mf}{p^1} - \frac{mf}{p^1} - r\right)\frac{t_1^2}{2} + (h^2\frac{mf}{p^2} - h^1\frac{mf}{p^1})\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{{t_1}^2}{2} - \beta\frac{{t_1}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{{t_1}^3}{6}\right\} + (C_2^2 - C_2^1)f]\alpha;$$

And $A_R(\alpha) = W - (W-V)\alpha = \frac{1}{(t_1+t_2)}[C_1^3 + klmft_1 + h^3\left(k\frac{mf}{p^3} - \frac{mf}{p^3} - r\right)\frac{t_1^2}{2} + h^3 \frac{mf}{p^3}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{t_1{}^2}{2} - \beta\frac{t_1{}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{t_1{}^3}{6}\right\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + C_2^3f + ldmft_2] - \frac{1}{(t_1+t_2)}[(C_1^3\text{-}C_1^2) + h^3\left(k\frac{mf}{p^3} - \frac{mf}{p^3} - r\right)\frac{t_1^2}{2} - h^2\left(k\frac{mf}{p^2} - \frac{mf}{p^2} - r\right)\frac{t_1^2}{2} + (h^3 \frac{mf}{p^3} - h^2 \frac{mf}{p^2}) \left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{t_1{}^2}{2} - \beta\frac{t_1{}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{t_1{}^3}{6}\right\} + (C_2^3 - C_2^2)f]\alpha$

The fuzzy average total cost function ($Tac\,\widetilde{(t_1,\ t_2)}$ is defuzzified by Signed Distance Method as follows,

$$Tac_{sd}\widetilde{(t_1,\ t_2)} = \frac{1}{2(t_1+t_2)}[C_1^1 + klmft_1 + h^1\left(k\frac{mf}{p^1} - \frac{mf}{p^1} - r\right)\frac{t_1^2}{2} + h^1 \frac{mf}{p^1}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{t_1{}^2}{2} - \beta\frac{t_1{}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{t_1{}^3}{6}\right\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + C_2^1f + ldmft_2] + \frac{1}{4(t_1+t_2)}[(C_1^2\text{-}C_1^1) + h^2\left(k\frac{mf}{p^2} - \frac{mf}{p^2} - r\right)\frac{t_1^2}{2} - h^1\left(k\frac{mf}{p^1} - \frac{mf}{p^1} - r\right)\frac{t_1^2}{2} + (h^2 \frac{mf}{p^2} - h^1 \frac{mf}{p^1})\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{t_1{}^2}{2} - \beta\frac{t_1{}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{t_1{}^3}{6}\right\} + (C_2^2 - C_2^1)f] + \frac{1}{2(t_1+t_2)}[C_1^3 + klmft_1 + h^3\left(k\frac{mf}{p^3} - \frac{mf}{p^3} - r\right)\frac{t_1^2}{2} + h^3 \frac{mf}{p^3}\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{t_1{}^2}{2} - \beta\frac{t_1{}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{t_1{}^3}{6}\right\} + \beta lmf\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^3}{6} - \frac{t_1^2}{2}t_2\} + C_2^3f + ldmft_2] - \frac{1}{4(t_1+t_2)}[(C_1^3\text{-}C_1^2) + h^3\left(k\frac{mf}{p^3} - \frac{mf}{p^3} - r\right)\frac{t_1^2}{2} - h^2\left(k\frac{mf}{p^2} - \frac{mf}{p^2} - r\right)\frac{t_1^2}{2} + (h^3 \frac{mf}{p^3} - h^2 \frac{mf}{p^2})\left\{\frac{(t_1+t_2)^2}{2} + \beta\frac{(t_1+t_2)^4}{12} - t_1t_2 - \frac{t_1{}^2}{2} - \beta\frac{t_1{}^4}{12} - \frac{\beta(t_1+t_2)^3}{6}t_1 + \beta(t_1+t_2)\frac{t_1{}^3}{6}\right\} + (C_2^3 - C_2^2)f] \quad (18)$$

Now, the necessary condition for the average total cost function of the system is minimized if equation (18)satisfies

$$\frac{\partial Tac_{sd}\widetilde{(t_1,\ t_2)}}{\partial t_1} = 0, \quad (19)$$

And $$\frac{\partial Tac_{sd}\widetilde{(t_1,\ t_2)}}{\partial t_2} = 0, \quad (20)$$

The solution, which might be called feasible solution of the problem, of the conditions (19) and (20) gives the optimal solutions of $t_1=t_1^*$ and $t_2=t_2^*$ which minimize $Tac_{sd}\ (t_1,\ t_2) = Tac_{sd}\ (t_1,\ t_2)^*$ provided they satisfy the sufficient conditions-

$$\frac{\partial^2 Tac_{sd}\widetilde{(t_1,\ t_2)}}{\partial t_1^2} \cdot \frac{\partial^2 Tac_{sd}\widetilde{(t_1,\ t_2)}}{\partial t_2^2} - \left(\frac{\partial^2 Tac_{sd}\widetilde{(t_1,\ t_2)}}{\partial t_1\,\partial t_2}\right)^2 > 0 \quad (21)$$

And $$\frac{\partial^2 Tac_{sd}\widetilde{(t_1,\ t_2)}}{\partial t_1^2} > 0 \quad Or, \quad \frac{\partial^2 Tac_{sd}\widetilde{(t_1,\ t_2)}}{\partial t_2^2} > 0 \quad (22)$$

However, it's difficult to solve the problem by inferring an explicit equation of the solutions from conditions (19) and (20). Therefore, the optimal service level t_1^* and the optimal cycle time

$t_1^*+t_2^*$ is solved by using the software LINGO 16.0. Moreover, it is also verified that the sufficient conditions of the optimality of the solutions t_1^* and t_2^* are satisfied (i.e. inequalities (21) and (22)) under certain conditions.

Similarly, the highest inventory level per unit time in fuzzy environment is given by

$$\widetilde{L_m} = \left(k\frac{mf}{\tilde{p}} - \frac{mf}{\tilde{p}} - r\right) t_1$$

$$=\left(k\frac{mf}{(p1.p2.p3)} - \frac{mf}{(p1.p2.p3)} - r\right) t_1 \qquad (23)$$

Defuzzified value of fuzzy number (L_m) by using Signed Distance Method is given by-

$$(\widetilde{L_m})_{sd} = \frac{1}{2}\left(k\frac{mf}{p1} - \frac{mf}{p1} - r\right) t_1 + \frac{1}{4}\left(\left(k\frac{mf}{p2} - \frac{mf}{p2} - r\right) - \left(k\frac{mf}{p1} - \frac{mf}{p1} - r\right)\right) t_1 + \frac{1}{2}\left(k\frac{mf}{p3} - \frac{mf}{p3} - r\right) t_1 - \frac{1}{4}\left(\left(k\frac{mf}{p3} - \frac{mf}{p3} - r\right) - \left(k\frac{mf}{p2} - \frac{mf}{p2} - r\right)\right) t_1 \qquad (24)$$

Numerical Solution

VI-A: For crisp model: To show the proposed technique, let's consider the accompanying input value in-

Table-1: Input Data

C_1	k	C_2	m	f	h	p	d	l	r	β
175	1.4	50	2565	6	1.2	11.6	1.3	0.15	2	0.01

The solution of the crisp-model is in Table-2

Table-2 : Result

t_1^*	t_2^*	$Tac(t_1, t_2)^*$	L_m^*
0.9168	0.5023	3816.822	484.7281

VI-B : For Fuzzy Model : Gives a chance to assume the parameters in fuzzy sense as : $\widetilde{C_1} = (150, 175, 200)$, $\tilde{p} = (8.6, 11.6, 14.6)$, $\widetilde{C_2} = (25, 50, 75)$, $\tilde{h} = (0.84, 1.2, 1.56)$, , where other parameters are unchanged. The solution of fuzzy model by Signed Distance Method is,

When $\widetilde{C_1}, \tilde{p}, \widetilde{C_2}$ and $\tilde{h}$ are all Triangular fuzzy number then the solution is given in-

Table-3: Result

t_1^*	t_2^*	$Tac_{sd}(t_1, t_2)^*$	L_m^*
0.9191	0.5040	3814.925	503.4081

(2) When $\widetilde{C_1}, \tilde{p}$, and $\widetilde{C_2}$ are Triangular fuzzy number then the solution is given in-

Table-4: Result

t_1^*	t_2^*	$Tac_{sd}(t_1, t_2)$^*	L_m^*
0.9031	0.4924	3828.560	494.6113

(3) When $\widetilde{C_1}$, and $\tilde{p}$ are Triangular fuzzy numbers then the solution is given in-

Table-5: Result

t_1*	t_2*	Tac_{sd} (t_1, t_2)*	L_m*
0.9031	0.4924	3828.560	494.6113

(4) When only $\widetilde{C_1}$ is Triangular fuzzy numbers then the solution is given in-

Table-6: Result

t_1*	t_2*	Tac_{sd} (t_1, t_2)*	L_m*
0.9168	0.5023	3816.822	484.7281

(5) When none of $\widetilde{C_1}$, $\tilde{p}$, $\widetilde{C_2}$ and $\tilde{h}$ is a Triangular fuzzy numbers then the solution is given in-

Table-7: Result

t_1*	t_2*	Tac_{sd} (t_1, t_2)*	L_m*
0.9168	0.5023	3816.822	484.7281

Comparison of Optimal Solutions is given in Table-8

Table-8: Comparison of Optimal Solutions

Model	Optimal value of t_1	Optimal value of t_2	Optimal value of Tac (t_1, t_2)	Optimal value of L_m
Crisp	0.9168	0.5023	3816.822	484.7281
Fuzzy	0.9191	0.5040	3814.925	503.4081

Sensitivity Analysis

Currently the sensitivity analysis of the optimal solution of the model for change system parameters C_1, k, C_2, m, f, h, p, d, l, r and β by -30%, -15%, +15%, +30% individually is analysed, keeping alternate parameters unaltered. The underlying information is taken from the numerical illustration.

Table-9 : Sensitivity Analysis

Parameters	Changed Value	*PCPV	t_1*	t_2*	Tac (t_1, t_2)*	L_m*
C_1=175	122.5	-30	0.8572	0.4795	3778.723	453.2304
	148.75	-15	0.8875	0.4911	3798.057	469.2162
	175	00	0.9168	0.5023	3816.822	484.7281
	201.25	+15	0.9454	0.5132	3835.066	499.8058
	227.5	+30	0.9731	0.5238	3852.839	514.4838
k=1.4	0.98	-30	0.0000	0.7685	4233.764	0.0000
	1.19	-15	1.7367	0.1635	3268.603	434.3065
	1.4	00	0.9168	0.5023	3816.822	484.7281
	1.61	+15	0.4117	0.6943	4121.946	332.3405
	1.82	+30	0.0184	0.7681	4233.477	19.0962
C_2=50	35	-30	0.8123	0.4622	3750.002	429.4780
	42.5	-15	0.8660	0.4828	3784.308	457.8487
	50	00	0.9168	0.5023	3816.822	484.7281
	57.5	+15	0.9653	0.5208	3847.801	510.3290
	65	+30	1.0115	0.5385	3877.443	534.8182

m=2565	1795.5	-30	1.1219	0.5797	2762.855	414.5397
	2080.25	-15	1.0329	0.5462	3155.231	442.4768
	2565	00	0.9168	0.5023	3816.822	484.7281
	2949.75	+15	0.8459	0.4753	4337.446	514.5294
	3334.5	+30	0.7875	0.4529	4854.907	541.6986
f=6	4.2	-30	0.9975	0.5325	2707.156	308.5458
	5.1	-15	0.9508	0.5150	3262.388	426.9689
	6	00	0.9168	0.5023	3816.822	484.7281
	6.9	+15	0.8911	0.4926	4370.720	542.0407
	7.8	+30	0.8708	0.4850	4924.238	599.0377
h=1.2	0.84	-30	1.0638	0.6151	3709.147	562.4372
	1.02	-15	0.9820	0.5509	3765.363	519.1984
	1.2	00	0.9168	0.5023	3816.822	484.7281
	1.38	+15	0.8633	0.4640	3864.551	456.4212
	1.56	+30	0.8183	0.4328	3909.253	432.6353
p=11.6	8.12	-30	0.7844	0.4104	3945.814	593.0900
	9.86	-15	0.8545	0.4581	3872.760	531.7821
	11.6	00	0.9168	0.5023	3816.822	484.7281
	13.35	+15	0.9743	0.5447	3771.299	446.3356
	15.08	+30	1.0244	0.5829	3735.587	416.1339
d=1.3	0.91	-30	0.1356	0.7619	3325.834	71.7164
	1.105	-15	0.6054	0.6614	3621.825	320.0902
	1.3	00	0.9168	0.5023	3816.822	484.7281
	1.425	+15	1.1212	0.3035	3944.362	592.7702
	1.69	+30	1.2193	0.0642	4005.503	644.6292
l=0.15	0.105	-30	0.9551	0.4762	2870.387	504.9487
	0.1275	-15	0.9363	0.4894	3343.782	495.0048
	0.15	00	0.9168	0.5023	3816.822	484.7281
	0.1725	+15	0.8968	0.5150	4289.504	474.1139
	0.195	+30	0.8760	0.5274	4761.822	463.1572
r=2	1.4	-30	0.9161	0.5024	3817.035	484.9041
	1.7	-15	0.9165	0.5024	3816.929	484.8161
	2	00	0.9168	0.5023	3816.822	484.7281
	2.3	+15	0.9172	0.5022	3816.716	484.600
	2.6	+30	0.9176	0.5022	3816.609	484.5518
β=0.01	0.0070	-30	0.9172	0.5048	3816.068	484.8918
	0.0085	-15	0.9170	0.5038	3816.446	484.8086
	0.0100	00	0.9168	0.5023	3816.822	484.7281
	0.0115	+15	0.9167	0.5011	3817.197	484.6501
	0.0130	+30	0.9153	0.4865	3821.674	483.8837

**PCPV = Percentage Change in Parameter Values.*

Observations

From the Table 9 the following can be closed:

(1) From the Table 9, for increasing of C_1, the optimal value of t_1* and t_2* increases slowly. With this effect, the total average cost Tac (t_1, t_2)* and the highest inventory level L_m* increases slowly.

(2) From the Table, when k<1 i.e. 0.98, the optimal value of t_1* and the highest inventory level L_m* becomes zero where the optimal value of t_2* increases. With this effect the total average cost Tac (t_1, t_2)* increases. Apart from this, for increasing k, the optimal value of t_1* decreases and the optimal value of t_2* increases rapidly. With this effect, the total average cost Tac (t_1, t_2)* increases and the highest inventory level L_m* decreases rapidly.

(3) From the Table, for increase in C_2, the optimal value of t_1* and t_2* increases slowly. By this effect, the total average cost Tac (t_1, t_2)* increases slowly and the highest inventory level L_m* increases rapidly.

(4) From the Table, for increase in m, the optimal value of t_1* and t_2* decreases slowly. With this effect, the total average cost Tac (t_1, t_2)* and the highest inventory level L_m* increases rapidly.

(5) From the Table, for increase in f, the optimal value of t_1* and t_2* decreases slowly. With this effect, the total average cost Tac (t_1, t_2)* and the highest inventory level L_m* increases rapidly.

(6) From the Table, for increase in h, the optimal value of t_1* and t_2*decreases slowly. With this effect, the total average cost Tac (t_1, t_2)* increases and the highest inventory level L_m* decreases slowly.

(7) From the Table, for increase in p, the optimal value of t_1* increases rapidly and the optimal value of t_2* increases slowly. With this effect, the total average cost Tac (t_1, t_2)* decreases slowly and the highest inventory level L_m* decreases rapidly.

(8) From the Table, for increase in d, the optimal value of t_1* increases and the optimal value of t_2* decreases rapidly. With this effect, the total average cost Tac (t_1, t_2)* increases slowly and the highest inventory level L_m* increases rapidly.

(9) From the Table, for increase in l, the optimal value of t_1* decreases and t_2* increases slowly. With this effect, the total average cost Tac (t_1, t_2)* increases rapidly and the highest inventory level L_m* decreases slowly.

(10) From the Table, for increase in r, the optimal value of t_1* increases and the optimal value of t_2* decreases slightly. With this effect, the increment of the total average cost Tac (t_1, t_2)* and the decrement of the highest inventory level L_m* is negligible.

(11) From the Table, for increase in β, the decrement of the optimal value of t_1* and t_2* is negligible. With this effect, the increment of the total average cost Tac (t_1, t_2)* is very slow and the decrement of the highest inventory level L_m* is negligible.

Conclusions

Here a genuine E. P. Q. Inventory Model is proposed and solutions are provided along affectability examination approach. Table-9 indicates when deterioration, production cost, holding cost is lesser, average cost function of the system decreases. Whereas it is also observed that lesser the population, lesser the demand and lesser the selling price, greater the demand. Here, a crisp model is produced then it changed to fuzzy model taking triangular fuzzy number and illuminated by Signed Distance Method. Decision maker may get the ideal outcomes as per his desire utilizing the result of this model. In future, the other sort of membership functions, for example, Parabolic Fuzzy Number (pFN), Generalised Fuzzy Numbers, Piecewise Linear Hyperbolic Fuzzy Number, Parabolic level Fuzzy Number (PfFN), Pentagonal Fuzzy Number and so forth can be considered to build the membership function and afterward that model can be effectively solved by Werner's Approach, Nearest Interval Approximation, Geometric Programming (GP) strategy, Nearest Symmetric Triangular Defuzzification (NSTD) technique, and so forth.

Limitations of the Study

In this proposed model of the inventory system, there were a few constraints, which are as follows:

- The inventory system includes just a single item and one stocking point.
- The proposed model is restricted here on the grounds that shortages are not permitted.

Future Scope

In future, researchers can extend this model by taking allowable shortages, two warehouses, stock dependent demand, permissible delay in payment, stochastic demand and inflation. Further, other sort of membership functions, for example, Parabolic Fuzzy Number (pFN), Generalised Fuzzy Numbers, Piecewise Linear Hyperbolic Fuzzy Number, Parabolic level Fuzzy Number (PfFN), Pentagonal Fuzzy Number and so forth can be considered to build the membership function and afterward that model can be effectively solved by Werner's Approach, Nearest Interval Approximation, Geometric Programming (GP) strategy, Nearest Symmetric Triangular Defuzzification (NSTD) technique, and so forth.

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