

Buyer's Optimal Policy for a Deterministic Inventory Model with Discounted Advance Payment and Trade Credit in a Supply Chain System

Rajesh Gupta

Prestige Institute of Management, Gwalior, India

Swati Agrawal

M B Khalsa College, Indore, India

Snigdha Banerjee

Devi Ahilya University, Indore, India

Abstract

An inventory model was investigated for a supply chain with a partial advance payment under discount and stated advance period where there is a single supplier providing a product to a single buyer. The situation under consideration is that the dominant player is the buyer who, while paying the advance decides upon the discount factor he would get on his purchase price for the units for which he pays the advance. The goal is to find optimal values for the buyer's cycle length, payment policy and the discount factor. A solution procedure was provided in order to obtain the optimal values of the decision variables. Numerical example is presented to demonstrate the solution procedure. Sensitivity analysis is carried out to examine the changes in the optimal values of decision variables as the model parameters change. It is found that optimal buyer's cycle length is affected by the rates of interest: both to be paid and earned by the buyer, optimal value of the net profit is affected by the buyer's purchase and selling prices as well as by the demand rate.

Key words: Supply Chain, Partial Advance Payment, Discount, Trade Credit, Inventory

JEL Classification: C61, E51

Paper Classification: Research Paper

Introduction

The classical Wilson's Economic Order Quantity (EOQ) inventory model assumes that the retailer pays his purchase price to the supplier as soon as items arrive in his stock. However in practice, at times this is not possible for the retailer. Often, supplier permits a specified period of interest-free delay in payment - called the credit period - to the retailer. This specified interest



free credit period (say, C days) denoted as 'net C ' in financial management - is used frequently by small businesses as a strong incentive for the buyer (Thangam 2012). Sometimes when the dues are to be cleared in n days at full price, a cash discount say $b\%$ is offered to the retailer if he clears the dues within a specified period say, $d(<n)$ days.

This business practice is denoted by "b/d, net n" (Brigham (1995, p. 741). During the credit period, the retailer can earn revenue by selling the goods and also earn interest on the revenue. Thus, it is desirable for the retailer to take advantage of the contract and to defer his payment till the end of the credit period. However, the supplier imposes a high rate of interest on the retailer for the unpaid amount if his dues are not cleared by the end of the credit period.

The benefits of trade credit policy as a marketing strategy are (i) increased sales (ii) new customers being attracted, for whom credit policy is a kind of price reduction (iii) reduction in the outstanding sales, as an established customers may pay more promptly in order to frequently utilize the credit period without payment of interest and also sometimes to get discount for early payments, (Teng 2002, Teng et al. 2014, Wu and Chan 2014, Zhang et al. 2014).

The influence of trade credit in payments on the optimal inventory system is a key issue of consequence in practical environments. Economic order quantity inventory models under trade credit have been considered widely in inventory literature since the seminal work by Goyal (1985). Trade credit models entailed widespread appeal and experienced vast applicability during the global economic recession of the last decade where it was necessary to create and enhance demand by deferring payment dates of the buyer.

In the current post recession supply chain business scenario, a common problem faced by the supplier is unilateral extension of credit period by influential stalwart buyers. Recognising the fact that this kind of forced extension of credit period may later result in supply deficit, some of the buyers are trying to arrange for financial assistance for the supplier from banks or other financial institutions at an interest. This is especially true of FMCG (Fast Moving Consumer Goods) sector manufacturing giants like Procter and Gamble, who have increased their credit period from 45 days to 75 days. Around 2013, P&G sought the co-operation of banks so that if the supplier paid the due fees to the banks, their dues would be paid within 15 days of delivery of the goods (<https://www.reuters.com/article/us-procter-suppliers-idUSBRE93G14Z20130417>, April 17, 2013). Reverse factoring is a financing instrument for supply chains where the supplier and buyer both pay fees to the bank, the buyer gains due to the longer credit period provided to him while the supplier's cash flow is better as his dues are settled earlier. However, this fee may add to the financial burden of the insecure supplier. This is a practical problem which needs to be addressed using mathematical methods.

On the positive side, there are some instances of buyers like Unilever who use their extra cash flow for expanding their business and in turn, extending some relief to suppliers in terms of higher orders volumes. (<http://ww2.cfo.com/supply-chain/2016/08/delaying-payments-can-help-suppliers/>). Despite these seemingly win-win solutions, there are downsides and constraints, hence triggering the search for better modes of managing the working capital of suppliers with support from the buyer (<http://ww2.cfo.com/supply-chain/2016/08/delaying-payments-can-help-suppliers/>). Sometimes, in order to quicken the receipt of payments, a common practice is that the supplier will try to motivate his clients to pay earlier by providing credit period as well as offer a discount (<https://primerevenue.com/understanding-early-payment-discount-terms/>). Goyal(1985) presented an EOQ inventory model under trade credit. He considers the lot-sizing problem with credit period for a buyer where the credit period may or may not exceed the length of the replenishment cycle. Later, many researchers have considered analytical inventory models



that consider permissible delays in payments for instance, Aggarwal and Jaggi (1995), Shin (1997), Jamal et al (2000), Salameh et al (2003), Teng et al (2005), Cheng et al (2010) and Chung (2012).. In all these papers, delay period was considered to be fixed. Some researchers assumed that in order to encourage the buyer to undertake an early settlement of accounts, the supplier puts forth an offer of a cash discount if the buyer pays his dues earlier. (Ouyang et al (2002), Ouyang et al (2005), Chang (2002), Goyal et al (2007), Sana and Chaudhuri (2008), and Yang (2010). Advance payment (prepayment or early payment) is a kind of reverse trade credit in which buyers pay the money before the delivery of goods. In some situations, suppliers offer substantial discounts on advance payment in order to attract substantial advance payments for sales. Partial advance payments are frequently encountered in international transactions. Quoting an IMF(2009) study, Zhang and Wang(2014) state that in about 19-22 percent of the international transactions, advance payments are made.

In the real world, there are ample instances of situations where partial or full advance payments are mandatory. For example, advance deposit required by caterers for food and beverage services, travel tickets, orders for customised high-cost equipment, maintenance and service agreements etc. For a specific product, if there are a limited number of suppliers in the market, the supplier may entail partial or full payment of the purchasing cost prior to delivery of goods, i.e., partial prepayment or full prepayment, respectively (Taleizadeh 2014a, b; Taleizadeh et al. 2011, 2013a, Gupta et al. (2009).

The present paper considers demand rate to be constant. Constant demand or uniform demand rate is commonly encountered for functional commodities or for products which have reached the maturity stage of their product life cycle. In the recent inventory literature, constant demand rate is discussed in inventory models with trade credit under different scenarios. Ouyang and Yang (2015) discussed an integrated inventory model with order-size dependent trade credit and constant demand. Marchi et al (2016) considered a single vendor and a single buyer two stage supply chain assuming that the buyer had better access to capital and may provide credit to the vendor to invest in his productivity. Kim and Sarkar(2017) considered the trade-credit policy, transportations discount when the demand is uniform.

Research Gap and Contribution

In most of the above mentioned articles, period of trade credit is considered to be fixed with fixed selling price or with cash discount - as decided by the supplier - for advance payment, which is in fact made during the credit period. However no payment is required before delivery of goods. Also, some articles consider advance payment scheme with trade credit without cash discount where either fixed or partial pre-payment is made as decided by the supplier. So far, discounted rate for making advance payment has not been considered to be decided by the buyer. In this paper, a payment policy was provided for the buyer which considers the above gap. Our contribution to the existing literature is to develop a payment policy in which the buyer has the option to make advance partial payment to the supplier for buying goods at a discounted rate. We consider the situation where the buyer can negotiate the discount and decide upon his proportion of advance payment accordingly for duration of advance payment and credit period specified by the supplier. The goal of considering such an inventory model is to obtain the optimal order quantity and optimal payment policy in terms of values of the proportions of payments to be made in two different phases: Advance payment and payment at the end of the credit period. Apart from optimal order quantity, optimal values of the proportions of the order quantity for which purchase price is to be paid in the two phases are obtained. Optimal value of discount and the total net profit of the retailer are also obtained.

The remaining part of the paper is organised as follows. Section 2 presents the notations and assumptions while the working of the model is described in section 3. Section 4 presents the mathematical formulations. Theoretical results and numerical example are presented in section 5 and 6 respectively, section 7 presents sensitivity analysis and section 8 concludes the paper.

Notations and assumptions

The notations and assumptions of the model are presented below:

Notations

D = Constant demand rate

P_s = unit selling price of supplier

P_R = unit selling price of the buyer. ($P_R > P_s$)

ρ = discount factor for advance payment $0 \leq \rho \leq 1$. Decision variable

ρ is such that ρP_R is the discounted purchase rate for advance booking

Note: The discount percent is $100(1 - \rho)$

h = holding cost per unit per unit time

A = cost of placing an order

IPR = interest rate for loan paid by retailer

IER = interest rate earned by retailer

MA = period of advance booking

MR = period of permissible delay in payment

T = cycle length. Decision variable

Q = order quantity

Note: $Q = DT$

A_1 = Proportion of ordered units for which advance payment is made. Decision variable

Net = the net profit per unit time

* With any variable indicates its optimal value

Assumptions

We assume that the payment can be made only at two epochs: MA time units before beginning of the season and at the end of the credit period.

The inventory system under consideration deals with a single item.

The planning horizon is infinite.

Shortages are not allowed.

Replenishment rate is infinite.

The buyer starts selling the goods as he gets them. This may involve negligible manufacturing time in case the buyer is a manufacturer and is purchasing the goods as raw material for manufacturing.

The accumulated earnings may be withdrawn only at epoch T . For all payments, the buyer has to take loan.

There is a huge penalty for not paying the complete dues by the end of the credit period.

If the buyer's dues are not cleared before the end of the replenishment cycle, the next replenishment will not be provided to him.

The Model

The model is developed from the point of view of a buyer while considering a single supplier-single buyer deterministic inventory model. The supplier provides trade credit to the buyer. If - as per his choice - the buyer makes some advance payment at the time stipulated by the supplier before the selling season begins, the supplier provides discount as decided by the buyer. We aim at maximising net profit for the buyer in this situation. The buyer places order for Q units M_A time units before the selling season starts while the ordered goods are delivered at the beginning of the season at epoch 0. The supplier provides credit period of length M_R to the buyer. As per the credit terms the buyer can make his payment in two parts:

- (1) An advance optional payment at epoch M_A for proportion, i.e., for (A_1Q) units at the discounted rate ρPS . Discount percentage is $(1-\rho) * 100$. $0 \leq A_1 \leq 1$; $0 \leq \rho \leq 1$.
- (2) A compulsory payment for the remaining proportion $A_2 = 1 - A_1$ i.e., for (A_2Q) units at the rate PS when the credit period ends at the epoch M_R . $A_1 + A_2 = 1$. No interest is to be paid for this delayed payment to the supplier.

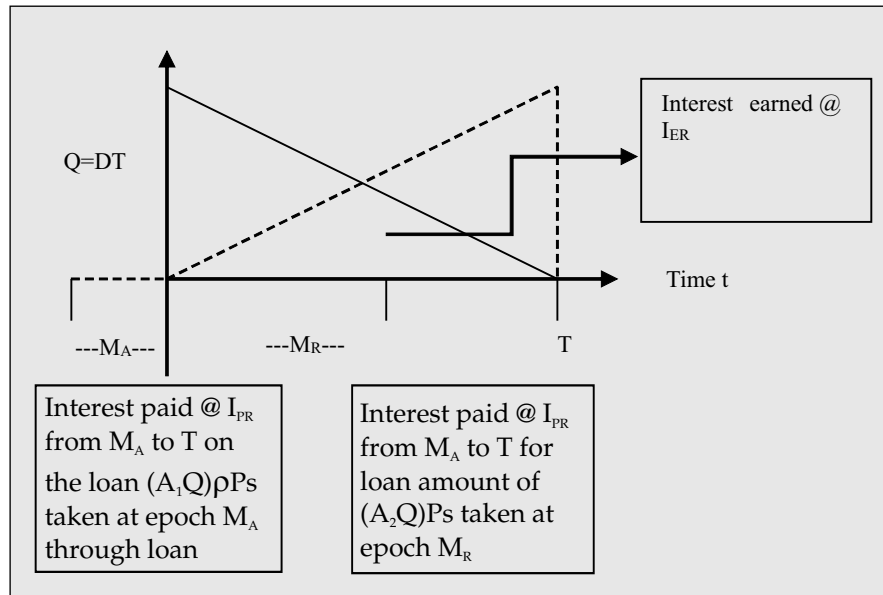
The buyer starts selling his goods from beginning of the cycle and invests the revenue earned as it accrues to earn interest at the rate I_{ER} . Payment to the supplier at M_A and M_R is made by borrowing the amount from financial institutions/ firms. The entire borrowed amount is repaid at T with interest at the rate I_{PR} .

Payment Policy: By assumptions 8 and 9, the cycle length T has to be chosen larger than the length of the credit period as the buyer tries to take full advantage of the credit period, is charged a huge penalty for not being able to pay by the end of the credit period and does not get the next replenishment if his dues are not paid before the end of the replenishment cycle.

In the paper, we thus only discuss the situation where the supplier makes it mandatory for the buyer to settle his payment completely at the end of the credit-period. For instance, this may in practice carry some stigma associated with future transactions.

We thus only consider the case when $M_R \leq T$, and where the buyer has to clear all suppliers dues at M_R . That is, the buyer takes full advantage of the credit period and cycle length exceeds the credit period. This mitigates the supply risk as the chance that the supplier falls short of finances or requires a loan for manufacturing the next lot is eliminated (See Figure 1).

Figure 1: Graphical representation of trade credit in inventory model under advance payment when $MR \leq T$



Mathematical formulation

The retailer’s net profit per unit time is given by

Net = $1/T$ ((a) Total revenue: sales revenue + Interest earned on sales revenue - (b) ordering cost - (c) holding cost - (d) purchase cost + interest payable) ,

where

(a) Total revenue earned per unit time = $\frac{P_R(DT)}{T} + \frac{P_R(DT)I_{ER}T}{2T}$

(b) Ordering cost per unit time = A/T

(c) Stock holding cost = $\frac{h(DT)T}{2}$

(d) Purchasing cost and Interest payable

I. Cost of A_1Q quantity paid at M_A , i.e., at the time of advance booking is:

Sum of the discounted cost for the A_1 units and the interest paid by the retailer for the amount paid through loan at the epoch M_A , till T , i.e., the end of the season, i.e.,

$$(\rho P_S)(A_1Q) + (T - M_A)(A_1Q)(\rho P_S)I_{PR}$$

II. Payment for A_2Q quantity paid at M_R , i.e., at the end of permissible credit period is the amount paid at M_R and the interest paid by the retailer for the amount paid at M_R through loan, i.e.,

$$P_S(A_2Q) + (T - M_R)(A_2Q)P_S I_{PR}; \text{ where } A_2 = 1 - A_1$$

In order to capture the role of A_1 and A_2 in the variation of Net and to concretize ideas and gain insights, in this regard we consider A_1 to be a concave function of ρ . To this end, we introduce constants a , b and c .

$$\text{Let } A_1 = ae^{-(b(\rho-c)^2)} ; b > 0 \tag{1}$$

$$\text{and } A_2 = 1 - A_1 \tag{2}$$

In the expression for Net presented above, on substituting from (a)-(d) and further substituting for A1 and A2 from (1) and (2) respectively, we get

$$\begin{aligned} \text{Net} &= P_R D - P_S D(1 - I_{PR} M_R) - \frac{A}{T} + ae^{-b(\rho-c)^2} P_S D((1 - I_{PR} M_R) - \rho(1 - I_{PR} M_A)) + \tag{3} \\ T &(-\frac{hD}{2} + \frac{I_{ER} P_R D}{2} - P_S D(1 - (1 - \rho)ae^{-b(\rho-c)^2}) I_{PR}) \end{aligned}$$

Theoretical results

For Net to be maximum, the first order (necessary) conditions are

$$\frac{\partial \text{Net}(T, \rho)}{\partial T} = 0, \frac{\partial \text{Net}(T, \rho)}{\partial \rho} = 0$$

and the second order (sufficiency) conditions are

$$\frac{\partial^2 \text{Net}(T, \rho)}{\partial T^2} < 0, \frac{\partial^2 \text{Net}(T, \rho)}{\partial \rho^2} < 0$$

$$\text{and } \begin{vmatrix} \frac{\partial^2 \text{Net}(T, \rho)}{\partial T^2} & \frac{\partial^2 \text{Net}(T, \rho)}{\partial T \partial \rho} \\ \frac{\partial^2 \text{Net}(T, \rho)}{\partial \rho \partial T} & \frac{\partial^2 \text{Net}(T, \rho)}{\partial \rho^2} \end{vmatrix} > 0$$

$$\text{i. e., } (\frac{\partial^2 \text{Net}(T, \rho)}{\partial T^2} * \frac{\partial^2 \text{Net}(T, \rho)}{\partial \rho^2} - (\frac{\partial^2 \text{Net}(T, \rho)}{\partial T \partial \rho})^2) > 0$$

Differentiating (3) with respect to T and equating to zero we get

$$\frac{\partial \text{Net}(T, \rho)}{\partial T} = \frac{A}{T^2} - \frac{hD}{2} + \frac{I_{ER} P_R D}{2} - P_S D(1 - (1 - \rho)ae^{-b(\rho-c)^2}) I_{PR} = 0 \tag{4}$$

On solving (4), we get

$$T^* = \frac{\sqrt{2Ae^{\frac{1}{2}b(\rho-c)^2}}}{\sqrt{e^{b(\rho-c)^2}(hD - I_{ER} P_R D) + 2P_S D(-a + e^{b(\rho-c)^2} + a\rho) I_{PR}}} \tag{5}$$

The optimal value of T for given MR must satisfy the condition $0 < M_R \leq T$.

Differentiating (4) with respect to T we get

$$\frac{\partial^2 \text{Net}}{\partial T^2} = -\frac{2A}{T^3} < 0 \tag{6}$$

$$\begin{aligned} \frac{\partial \text{Net}(T, \rho)}{\partial \rho} &= -ae^{-b(\rho-c)^2} P_S D[(1 - 2b\rho(\rho - c))(1 + I_{PR}(M_A + T)) + 2b(\rho - c)(1 + \\ I_{PR}(T - M_R))] &= 0 \tag{7} \end{aligned}$$

$$\rho^* = \frac{2b(1+c+c(M_A+T))I_{PR}-(T-M_R)I_{PR} - \sqrt{-2b^2(1+c+c(M_A+T))I_{PR}-(T-M_R)I_{PR})^2 - 8b(1+(M_A+T))I_{PR}(-1+2bc - I_{PR}(M_A+T) - 2bcI_{PR}(T-M_R))}}{4b(1+I_{PR}(M_A+T))} \tag{8}$$

The optimal value of ρ must satisfy the condition $0 \leq \rho \leq 1$.



The second-order derivative of Net with respect to ρ is given by

$$\frac{\partial^2 \text{Net}}{\partial \rho^2} = -2abe^{-b(\rho-c)^2} P_5 D[(1 - 2b(\rho - c)^2)(1 + I_{PR}(T - M_R)) + (2c - 3\rho + 2b(\rho - c)^2)(1 + I_{PR}(M_A + T))] \quad (9)$$

In the following theorem, we find a wide sufficient condition for concavity of Net with respect to ρ .

Theorem: Net is a concave function of ρ if $c - \frac{1}{\sqrt{b}} < \rho < \min(2c/3, c + \frac{1}{\sqrt{b}})$

$$\frac{\partial^2 \text{Net}}{\partial \rho^2} = -2abe^{-b(\rho-c)^2} P_5 D[(1 - 2b(\rho - c)^2)(1 + I_{PR}(T - M_R)) + (2c - 3\rho + 2b(\rho - c)^2)(1 + I_{PR}(M_A + T))]$$

Since $T > MR$, the first expression inside the bracket is positive if $1 > 2b(\rho - c)^2$,

$$\text{i.e., if } c - \frac{1}{\sqrt{2b}} < \rho < c + \frac{1}{\sqrt{2b}}$$

The second expression inside the bracket is positive if $(2c - 3\rho + 2b(\rho - c)^2) > 0$, i.e., if

$2b(\rho - c)^2 > 3\rho - 2c$. If $\rho < 2c/3$, then the inequality holds.

Hence, a wide sufficient condition for $\frac{\partial^2 \text{Net}}{\partial \rho^2} < 0$ is that $c - \frac{1}{\sqrt{b}} < \rho < \min(2c/3, c + \frac{1}{\sqrt{b}})$.

(Hence Proved)

Algorithm

Based on the above discussion, we develop the following solution algorithm to determine an optimal solution for the model.

Step 1: Initialise values of the all the parameters. Initialise a guess value ρ_0 of ρ .

Step 2: Set $i = 1$.

Step 3: Compute T_i^* from equation (5).

Step 4: Compute ρ_{i+1}^* from equation (8) after substituting the value of T_i^* from step 2.

Step 5: $i = i + 1$. Repeat the process till the values of T_i^* , ρ_{i+1}^* stabilizes.

Step 6: Obtain the optimal value of Net from equation (3).

Step 7: Obtain the optimal value of A_1 as A_1^* by putting optimal value of ρ^* and other parameters from equation (1) and hence optimal A_2^* from equation (2).

Step 8: Stop.

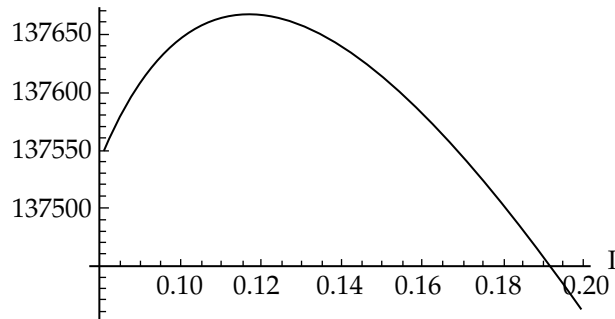
Numerical Example

We consider the following values for the input parameters in proper units.

$D = 10000$ units/ year, $h = \text{Rs. } 2/\text{ unit/year}$, $A = \text{Rs. } 100/\text{ order}$, $IER = \text{Rs. } 0.07/\text{ year}$, $IPR = \text{Rs. } 0.09/\text{Year}$, $PS = \text{Rs. } 5/\text{ unit}$, $PR = \text{Rs. } 18/\text{ unit}$, $a = 0.5$, $b = 2$, $c = 0.9$, $MR = .08$ year, $MA = 0.04$ year.

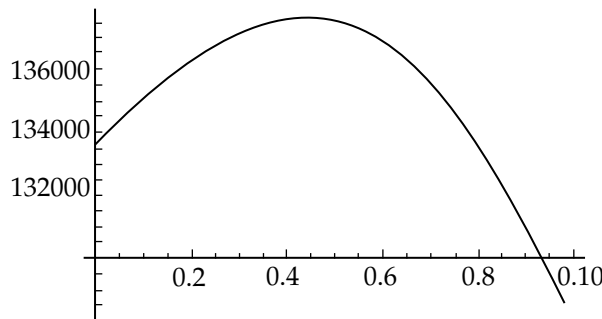
It can be easily seen from Figure 2 that Net is a concave function of T.

Figure 2: Net versus T



It can be easily seen from figure (3) that Net is a concave function of ρ .

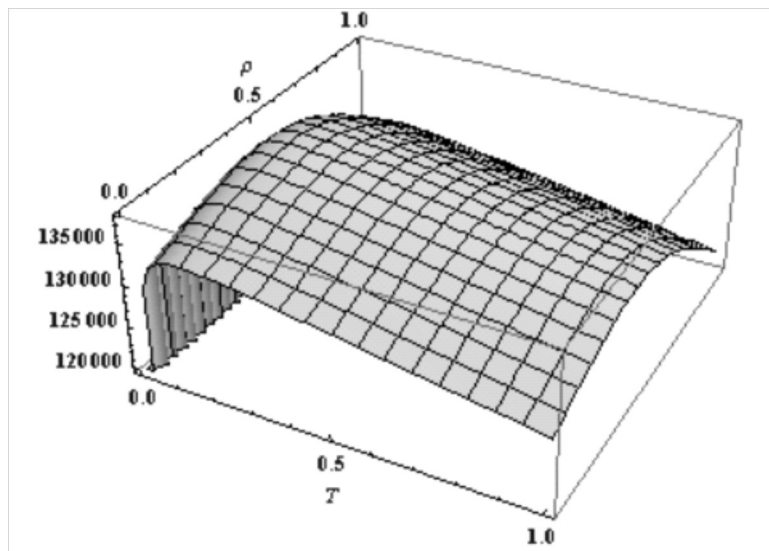
Figure 3: Net versus ρ



Further, from figure 4, we see that for the set of parameters considered, Net is joint concave function of T and ρ .

Plot of Net w.r.t. T and ρ

Figure 4: Net versus T and ρ



Using algorithm developed in section 5, we get the optimal values of decision variables as follows

$T^* = 0.116$ years $>$ MR (= .08 year), $\rho^* = 0.443$, $Q^* = 1164.47$ units and Net* = Rs. 137668. (See figure 4)

Sensitivity Analysis

Decision making situations are not devoid of uncertainties and hence, change may occur in parameter value and the implications of this need to be examined. Sensitivity analysis is a useful instrument in such situations. For the numerical example of the preceding section, sensitivity analysis was conducted to examine the change in the optimal solution due to changes in the values of model parameters by changing each of the parameters by -50 per cent, -25 per cent, +25 per cent and +50 per cent. Keeping the values of all other parameters at their original values, we change one parameter at a time. The results of the sensitivity analysis with respect to economic parameters are shown in Table 1 while of model parameters are given in Table 2.

Table 1: Sensitivity of the optimal solution with change in the economic parameters

Changing parameter	per cent Change in parameter value	T*	ρ^*	A1*	Net
$I_{PR} = 0.09$	-50%	15.404	0.543	0.439	0.089
	-25%	6.879	0.271	0.219	0.041
	25%	-5.701	-0.271	-0.219	-0.036
	50%	-10.528	-0.541	-0.438	-0.068
$I_{ER} = 0.07$	-50%	-16.292	-0.002	-0.002	-0.243
	-25%	-9.225	-0.001	-0.001	-0.126
	25%	12.764	0.001	0.000	0.142
	50%	32.122	0.004	0.003	0.304
$P_R = 18$	-50%	-16.292	-0.002	-0.002	-65.617
	-25%	-9.225	-0.001	-0.001	-32.814
	25%	12.764	0.001	0.001	32.829
	50%	32.122	0.004	0.003	65.678
$P_S = 5$	-50%	15.404	0.002	0.002	14.918
	-25%	6.879	0.001	0.001	7.456
	25%	-5.701	-0.001	-0.001	-7.451
	50%	-10.527	-0.001	-0.125	-14.897
A = 100	-50%	-29.289	-0.003	-0.003	0.365
	-25%	-13.398	-0.002	-0.001	0.167
	25%	11.804	0.001	0.001	-0.147
	50%	22.475	0.002	0.002	-0.280
h = 2	-50%	76.226	0.009	0.007	0.540
	-25%	22.998	0.002	0.002	0.233
	25%	-13.580	-0.002	-0.001	-0.196
	50%	-22.802	-0.002	-0.002	-0.368

Table 2: Sensitivity of the optimal solution with respect to the model parameters

Changing parameter	% Change in parameter value	T*	ρ^*	A1*	Net
a = 0.5	-50%	-2.685	0.000	-49.971	-3.312
	-25%	-1.370	0.000	-24.957	-1.656
	25%	1.429	0.000	25.072	1.657
	50%	2.922	0.000	50.087	3.314
b = 2	-50%	1.960	-46.653	-2.216	2.301
	-25%	0.718	-17.413	-1.005	0.859
	25%	-0.480	11.871	0.873	-0.585
	50%	-0.829	20.626	1.652	-1.015
c = 0.9	-50%	5.686	-65.757	27.143	6.287
	-25%	2.563	-30.412	16.065	2.959
	25%	-1.991	24.828	-21.102	-2.449
	50%	-3.409	44.155	-44.854	-4.278
$M_A = 0.04$	-50%	0.000	0.181	0.146	0.009
	-25%	0.000	0.090	0.073	0.005
	25%	0.000	-0.090	-0.073	-0.004
	50%	0.000	-0.180	-0.146	-0.009
$M_R = 0.08$	-50%	0.000	0.365	0.295	-0.088
	-25%	0.000	0.183	0.148	-0.044
	25%	0.000	-0.183	-0.148	0.044
	50%	#	#	#	#
D = 10000	-50%	41.421	0.005	0.004	-50.258
	-25%	15.471	0.002	0.002	-25.145
	25%	-10.557	-0.001	-0.001	25.165
	50%	-18.350	-0.002	-0.002	50.344

Note: '#' indicates non feasible value of T*, ρ^* , A1*, Net. The values in bold face show significant change

Table 1 leads to the following observations:

- Increases in PR by 50 per cent results in about 32 per cent increase in T* and 66 per cent increase in Net profit, while 50 per cent decrease in PR results in about 16 per cent decrease in T* and 66 per cent decrease in Net profit.
- Increase in PS by 50 per cent results in decrease in optimal time by 11 per cent and decrease in optimal profit about 15 per cent.
- Increase in IER by 50 per cent results in about 32 per cent increase in T* whereas surprisingly, this does not significantly affect the net profit. A 50 per cent decrease in IPR results in more than 15 per cent increase in T*.
- It is seen that the optimal cycle length T* is highly sensitive to changes in parameters A and h. Decrease in A by 50 per cent decreases T* by 29 per cent whereas increase in A by 50 per cent increases T* by 22 per cent. Decrease in h by 50 per cent increases T* by a massive 76 per cent while increase in h by 50 per cent reduces T* by about 23 per cent.

Table 2 leads to the following observations:

- As the demand D increases and decreases by 50 per cent, the optimal cycle time T decreases

and increases about 18 per cent and 41 per cent respectively, while the Net increases and decreases by about 50 per cent.

- It is seen that the optimal discount rate ρ^* is more sensitive to changes in parameters b and c. A 50 per cent decrease in the value of b results in 46 per cent decrease in ρ^* while 50 per cent increase in the value of b results in 20 per cent increase in ρ^* . Similarly, just a 25 per cent decrease in the value of c results in 30 per cent decrease in ρ^* while 25 per cent increase results in about 29 per cent increase in ρ^* .
- It is also observed that a and c are parameter that must be chosen with care as increase and decrease in these parameters result in significant change in $A1^*$. A 50 per cent reduction in 'a' results in about 50 per cent decrease in $A1^*$ while a 50 per cent increase in the value of 'a' results in a 50 per cent increase in $A1^*$. Similarly 50 per cent decrease in the value of c results in 27 per cent increase in $A1^*$ while 50 per cent increase in the value of c results in 45 per cent decrease in $A1^*$.
- As changes in MA and MR do not show significant changes in the values of quantity ordered by the buyer or on the net profit of the buyer, the buyer can agree to supplier's negotiation for a longer advance period and shorter credit period for the buyer thus ensuring supplier's cash flow and reduction in supply risk. This policy hence seems to be a viable policy.

Conclusion

A buyer's payment policy for supplier with permissible delay in payment with a plausible optional partial advance payment by the buyer at discounted price as decided by the buyer has been discussed. It was found that increasing the retailer's selling price results in significant increase in the order quantity as well as net profit while increase in supplier's selling price results in a relatively less but significant decrease in both order quantity as well as net profit. Other factors that result in significant change in optimal time are the rates of interest to be paid by and earned by the retailer. Change in the time for advance payment affects the negotiable discount rate as well as the proportions of payments in the two phases to some extent. Changes in the parameters a, b, and c of the function defining proportion of advance payment by the retailer affect all the decision variables. Especially the effect of change in c and b on discounted value of supplier price; c and a on the payment proportions is highly significant, while the percentage change caused to net profit is relatively small. Hence, the buyer can exploit this flexibility in selecting these parameters without affecting his optimum net profit. The change in demand rate results in significant changes in both - the net profit and the cycle time. Hence, demand rate must be estimated with care and the buyer should try to increase his demand rate in order to increase his profit. Further, increase in duration of advance payment by the supplier will not result in significant reduction in the buyer's net profit as the supplier has to provide increased discount rate.

Limitations of the Study and Future Scope

In the present study of the inventory model, there are some limitations, which are as follows:

1. This inventory model reduces the supplier risk of non-payment up to a great extent, but this study does not guarantee the elimination of supplier risk.
2. As prompted by the situation under consideration, this study is only limited to the case when the trade credit offer does not exceed the length of the replenishment cycle. Also the buyer has to clear all the dues of the supplier at MR.
3. In this study, no shortages are allowed.

The current study can be extended in several ways. In future, price dependent demand and payment allowed at T or $MR > T$ may be considered to make the model more practical.

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Authors' Profile

Rajesh Gupta is an Assistant Professor in Department of Mathematics, Prestige Institute of Management, Gwalior, India. He obtained M.Sc. degree from Devi Ahilya University, Indore, India. He is currently pursuing research in Statistics from School of Statistics, Devi Ahilya Vishwavidyalaya, Indore, India. His major research interest includes inventory models related to supply chain. He has presented papers in various national and international conferences.

Swati Agrawal is Assistant Professor in Department of Mathematics, MB Khalsa College, Indore, India. She obtained her B.Sc. and M.Sc. degrees from Devi Ahilya University, Indore, India. She has published papers in Journals like *International Transactions in Operational Research*, *International Journal of System Sciences* and *Applied Mathematical Modeling*. Her area of research is Inventory Management of deteriorating items and multiple warehouses. Her current interests include inventory models for freshness dependent demand.

Snigdha Banerjee is a Professor at the School of Statistics, Devi Ahilya University, Indore, India. She has published papers in the fields of operations research, bioinformatics and design of experiments in journals like *Journal of the Operational Research Society*, *International Transactions in Operational Research*, *Annals of the Institute of Mathematical Statistics*, *Applied Mathematical Modeling*, *International Journal of Production Economics*, *Computers and Industrial Engineering*, *Mathematical and Computers Modeling*, *Sankhya*, *Proteomics*, etc. She has also authored a book and has been instrumental in the development of a RDBMS software package. Her current interest is mostly on various aspects of Inventory management.