



## Effect of Demonetization & Implementation of GST on Dynamical Behaviour of Indian Stock Market

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### Abstract

The Indian economy has witnessed a few major changes during the last four years that include demonetization and implementation of GST. Here it is examined whether Indian share market developed any chaotic behaviour due to such major changes in economic front of the country by analysing the CNX Nifty data of the last four years with the help of Grassberger and Procaccia algorithm. The same is investigated for the Reliance Industries Limited, one of the major stocks of Nifty and is currently the largest company of the country in terms of market capitalisation. Findings indicate that some sort of chaotic nature is present in both CNX Nifty and stock price of the Reliance Industries during the last few years.

**Keywords:** Nifty, Reliance Industries Limited, Chaos, Strange Attractor, Demonetization, GST

**JEL:** E44

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### Introduction

Stock markets are considered as real examples of complex dynamic systems. The time variation of stock index usually exhibits a constant background of small up and down price movements. It also reveals a few sudden large changes and the big price/index jumps become more common as the turbulence of the market grows. Empirical evidences suggest the presence of fractals in time series of stock market price movements (Evertsz, 1995; Matia, Ashkenazy & Stanley, 2003). In the present investigation, whether the stock price movements in India during the last four years exhibits any fractal behavior or not have been examined.

A little more than four years back, amid great expectations the National Democratic Alliance (NDA) under the leadership of Sri Narendra Modi swept into power in India with the largest mandate in modern history (after 1984) of the country. Corporate, financial experts and investors were expecting wide ranging reforms, which would lead to growth. The Indian stock market was also cheered up with the formation of NDA government and experienced new all-time highs during the present tenure of the NDA government. During the first year of the rule of the new government, Indian economy showed sign of recovery with the growth rate being 6.9 per cent for the 2014 fiscal year from 4.5 per cent in the 2013 fiscal year though some economists ascribed the

jump in the growth rate on the fall in global crude oil price and the changed way of computing the GDP.

After few months, concerns were mounting over the performance of Indian stock market. It appears that the falling oil prices, worries about China's economy and the crisis in Syria had walloped stock markets. Besides, the world economy remained sluggish and hardly had the growth engine it once was. All this led to a volatile start of the market in the year 2016. Then, at the end of 2016, demonetization was implemented. In demonetization process, a running series of currency are not recognized as a legal tender from a certain period of time and therefore the series of currency are not acceptable as valid currency after that time. There is also no guarantee that the withdrawn money will be replaced with a new currency unit. The Government of India declared on 8th November 2016 that from the midnight of that day onward 500- and 1000-rupee bank notes will not be legal tender. Though initiative was taken to replace the sucked-out money with new currency unit but it had taken a considerable period of time. Due to demonization and Trump's victory in US presidential election, the stock market became completely bearish. The BSE Sensex lost 1600 points at open, nifty dropped 541.30 points. But slowly the market recovered. Note that unlike the previous instances of demonetization in several countries, where all of them were done when their economies were having major problems like hyper-inflation in Germany in the 1920s, this is the first instance that a perfectly healthy economy attempted to target black money.

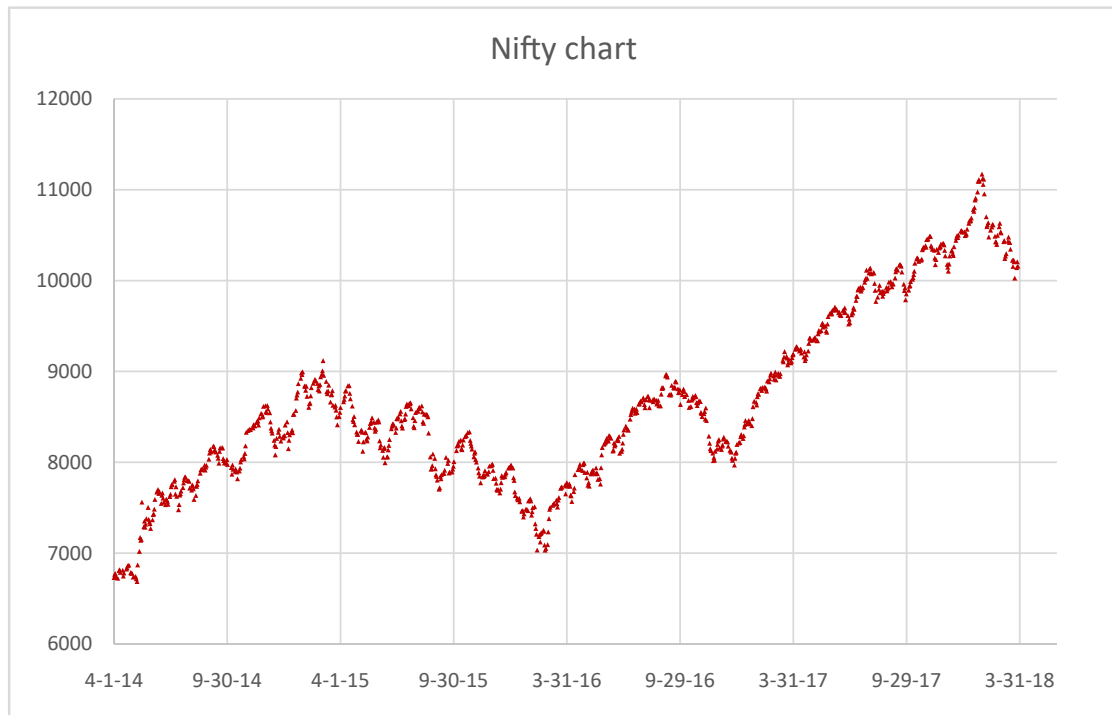
In the following year i.e. in 2017, another major economic event took place, the so called Goods & Services Tax (GST) was launched on July 2017 which was awaited for a long time. It is the biggest tax reform in Indian history since Independence as it brought all (or rather nearly all) indirect taxes under one uniform tax system. Therefore, GST is considered as a unified indirect tax structure. The economic growth of a country is supposed to be enhanced with the introduction of the GST. So far more than 150 countries have implemented GST. The Vajpayee government in 2000 first proposed the idea of GST in India. The Lok Sabha on 6th May 2015 passed the constitutional amendment for the GST. Several countries followed single GST system while a few countries opted for dual GST system where tax is imposed by central and state both. India has also adopted dual system of GST that contains CGST and SGST respectively imposed by Central and State Governments. Several authors studied that there would be huge impact on Indian economy definitely after its implementation. However, there was a huge hue and cry against its implementation at the initial stage and some fluctuations were noticed in stock market index.

It is generally believed that the mentioned unprecedented economic events in the history of the country since its independence led to a large volatility in almost all the economic sectors. The behaviour of one of the major Indian stock indices, the so-called Nifty, over the last four years is shown in Figure 1. It can be seen from Figure 1 that the stock index gradually rises to higher levels (based on country's growth story as well as impact of global stock market movement) but also many bumps with sudden large variations have been noticed on the growth road of the market. It is further revealed from the Figure 1 that after the announcement of demonetization the stock market went down but recovered quickly on its growth path. However, no major (sudden) change in Nifty due to implementation of GST has been noticed.

The movement of stock market itself is very complicated, it depends on large number of factors. Introduction of additional major events in economy is expected to affect the movement of stock market even at higher degree of complexity. In such complicated system, often chaos

develops. It is known that small changes in the initial conditions lead to drastic changes in the output/final results of a non-linear system. Under the circumstances, in this work it will be investigated whether Indian stock market developed any chaotic behaviour or not. For this purpose, the CNX Nifty high price data over last four years examined for any non-linearity taking the help of Grassberger and Procaccia (GP) algorithm (Grassberger & Procaccia, 1983). The GP algorithm is widely used to measure the fractal dimensions of a strange attractor from a univariate time series.

The high price data of Reliance Industry, one of the largest company of the country and a major stock of CNX Nifty will also be analysed for chaotic behaviour. For the analysis, the daily high price of CNX nifty and Reliance Industries over a period of four years from 01-04-2014 to 31-03-2018 are considered.



**Figure: 1 Time Variation of daily high price of NIFTY during 1st April 2014 to 31st March 2018**

### **A Brief Survey of Dissipative Dynamical System**

A dynamical system may be regarded as a deterministic mathematical instruction for developing the state of system over the course of time. Dynamical system theory is an area of mathematics that is used to describe the behaviour of complex dynamical system usually by employing the differential equations or difference equations.

The time variable in dynamical system, which may either be a continuous or take discrete integer values. According to their mathematical expressions - discrete or continuous, dynamical systems are categorised in two types. From technical point of view, a dynamical system can be regarded as a smooth action of the reals or integers on another object. Action of the reals on the system is stated a continuous dynamical system, and action of the integers on the system is called

discrete dynamical system. An example of a dynamical system in which time is a continuous variable is a system of  $N$  first-order, autonomous, ordinary differential equations,

$$\begin{aligned}\frac{dq^{(1)}}{dt} &= H_1 (q^{(1)}, q^{(2)}, q^{(3)}, \dots, q^{(N)}), \\ \frac{dq^{(2)}}{dt} &= H_2 (q^{(1)}, q^{(2)}, q^{(3)}, \dots, q^{(N)}), \\ &\vdots \\ \frac{dq^{(N)}}{dt} &= H_N (q^{(1)}, q^{(2)}, q^{(3)}, \dots, q^{(N)}),\end{aligned}$$

where  $q$  is a  $N$  dimensional vector space. In principle, the above equations can be solved to obtain the future system state  $q(t)$  for  $t > 0$  from any initial state of the system  $q(0)$ . Let  $V(t)$  be the volume of the region enclosed by the surface  $S_t$  and  $v(0)$  be the volume of the region enclosed by the surface. If then the system is called conservative. If  $V(t) = V(0)$  the volume shrinks, then the system is called dissipative. In dissipative systems trajectories are attracted to a fixed point. This is called attractor (Ott, 1993). Dynamical systems with continuous-time variable passage along a continuous trajectory in phase space whereas dynamical systems with discrete time variable are described by discrete points in phase space.

A first – order, autonomous discrete dynamical system for  $x_n \in \mathbb{R}^d$  has the form

$$x_{n+1} = f(x_n)$$

where  $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$  and  $n \in \mathbb{Z}$  is a discrete time variable.

A linear discrete dynamical system has the form

$$x_{n+1} = Bx_n$$

where  $B$  is a linear transformation on  $\mathbb{R}^d$ . The behavior of even one-dimensional discrete dynamical systems may be complicated and difficult to analyze. The series of daily high prices of CNX nifty is an example of discrete dynamical system.

## Chaotic Dynamics

A large number of laws of the nature are non-linear in the sense that while describing the laws by differential equations, the variables and their time derivatives are coupled in non-linear manner. Analytical solution is hardly found in nonlinear systems. Instead, solutions are generally studied in the phase space for a complete description of states of the system.

An important observation is that for nonlinear dynamical systems even small perturbations affect the solutions of system deeply. While studying simple mathematical model of weather with the aid of a computer, Lorentz first noticed that the results are very sensitive to initial conditions (Lorentz, 1963). Li and Yorke coined the term “chaos” for dynamical systems that are highly sensitive to initial conditions (Li & Yorke, 1975). Small changes in initial conditions, for instance, those due to rounding errors in numerical computation even produce widely deviating results for non-linear dynamical systems. Consequently long-term prediction of the behaviour of such

systems are in general not possible though these systems are deterministic as they are governed by differential equations. Such a behaviour is known as deterministic chaos, or simply chaos. The dynamical systems which exhibit such kind of behaviour are called chaotic systems. A point to be noted that within the apparent randomness of chaotic complex systems, there may be underlying patterns such as self-similarity.

Chaos has been found in many natural systems including physical science, life science, environmental science, economics, commerce, geography, meteorology, anthropology, sociology, computer science, engineering etc. The spontaneous occurrence of chaos is also noticed in systems with artificial components, such as road traffic.

### Strange Attractor and Fractal Dimension

The trajectory of a dynamical system in phase space may tend towards a particular point or orbit in the course of dynamical evolution on which the system comes to rest. Such point(s) of the phase space of the system are called attractor(s) of the system. The attractors are insensitive to the initial conditions of the system which means that in the long term a dynamical system exhibits a particular pattern which is not obvious in short term. Mathematically, neighboring states of a system approach asymptotically towards a set of state(s) which are attractor(s) of the system. The dimension of attractors may not be integers always. If dimension of attractors fall between the integers, such geometrical objects are called as fractals. Usually dimension of fractals exceeds its topological dimension. When an attractor is fractal, it is called a strange attractor. The two-dimensional Henon map is an example of a strange attractor,

$$y_{n+1}^{(1)} = A - \left(y_n^{(1)}\right)^2 + By_n^{(2)},$$

$$y_{n+1}^{(2)} = y_n^{(1)}$$

for  $A = 1.4$  and  $B = 0.3$ . Fractal dimension of the attractor is equivalent to .

### Source of the Data

In the present work the long run relationship between Nifty and Reliance Industries Limited for the period 1<sup>st</sup> April 2014 to 31<sup>st</sup> March 2018 have been studied using the daily high price data. Data has been collected covering the period from just before the formation of the new government to aftermath of GST implementation.

The Nifty and Reliance Industries high price data have been collected from the website of NSE ([www.nse.com](http://www.nse.com)).

### Methodology

#### Computation of Point Wise Dimension

There are several approaches for evaluating fractal dimension of time series. All techniques follow a common structure, a certain numerical feature (P) of the time series or the data is figured out as a function of "scale," say  $\varepsilon$  and then an asymptotic power law  $Q(\varepsilon) \propto \varepsilon^b$  is derived or postulated as the scale becomes infinitesimally small ( $\varepsilon \rightarrow 0$ ). In such a situation the scaling exponent will be a linear function of the fractal dimension D. In this paper, the fractal dimension of strange attractor in phase space (Theiler, 1990) of the

high prices of NIFTY and Reliance Industry Limited are estimated employing the popular GP algorithm. Note that the movement of high prices of major Indian oil companies and nifty index have very recently been investigated applying the GP Method (Sarkar, Paul & Majumder, 2015).

Let there be a series of  $N$  points in some metric space with coordinates  $X_i$ , where the suffix  $i$  varies from 1 to  $N$ . The modulus of distance between pair of points  $i$  and  $j$  is  $|X_i - X_j|$ . The correlation integral  $C(r)$  of the series essentially means the number of points which fall within a certain distance, say  $r$  from one another. Grassberger and Procaccia have demonstrated that the correlation integral acts as a power law for dissipative dynamical systems that exhibits chaotic feature with a strange attractor in the phase space. The correlation dimension which is the exponent of power law, is closely related to the fractal dimension. A dynamical system of difference equation designates the development of some variable over time. Let the magnitude of this variable in period  $t$  be denoted as  $X_t$ . The time index takes on discrete values and typically runs over all integer numbers e.g.,  $\dots, -2, -1, 0, 1, 2, \dots$ . The variable takes values in some metric space  $(X, d)$ , where  $d$  denotes the metric on  $X$ .  $X$  is referred as state space.

A difference equation in most general form is

$$F_N(Y_t, Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}, t) = 0 \quad (1)$$

where  $F_N$  where  $F_N$  is given function. Here the variable  $Y_t$  is dependent variable and is  $n$ -vector. Mathematically  $Y_t \in \mathbb{R}^n, n \geq 1$ .  $Y_t$  is the state of the system and its state space,  $n$  is the dimension of the system. The order of the difference equation is just the difference between the largest and the smallest time index of the dependent variable which equals to  $p$  in the formulation (1),  $p \geq 1$ . The time index appears explicitly as an argument of the function  $F_N$  in the above stated difference equation.

Unique solution of (1) for may be possible:

$$Y_t = f(Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}, t) \quad (2)$$

When starting values  $y_0, y_{(-1)}, y_{(-2)}, \dots, y_{(-p+1)}$  for  $Y_0, Y_{(-1)}, \dots, Y_{(-p+1)}$  are provided all subsequent values of  $Y_t$  can be uniquely determined from the above equation (2) by iteratively inserting into equation (2).

A solution to the equation (1) is a function  $Y: \mathbb{Z} \rightarrow \mathbb{R}^n$  such that  $Y(t)$  accomplishes the difference equation (1) like

$$F_N(Y(t), Y(t-1), Y(t-2), \dots, Y(t-p), t) = 0 \text{ holds for all } t \in \mathbb{Z} \quad (3)$$

The analysis judges the existence and uniqueness of a solution to a given difference equation and characterizes the set of all solutions.

Classical geometry is based on integer dimensions while fractal geometry deals with non-integer dimensions. Fractals are objects that are too irregular to fit into traditional geometrical figures. Examples of fractals include clouds, mountain ranges, landscapes, coastlines and several vegetables such as cauliflower and broccoli. A basic feature of fractals is self-similarity; it is a geometrical figure or image in which a pattern/block repeats itself on an ever diminishing/magnifying scale. It looks similar at all levels of magnification. The dimension is a relevant feature

of fractals which is a statistical quantity that gives an indication of how completely a fractal appears to fill space.

Let us consider a long-time series  $Y_i$  consisting of  $N$  points having the following characteristics

$$\{\vec{Y}_i\}_{i=1}^N \equiv \{\vec{Y}(t + i\tau)\}_{i=1}^N \tag{4}$$

where  $\tau$  is an arbitrary but fixed time increment or time delay.

As mentioned earlier the correlation integral reflects the probability of closeness of the states at two different times. By definition the correlation integral for any positive is given below

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i \neq j}}^N \theta(r - \|\vec{Y}_i - \vec{Y}_j\|) \tag{5}$$

where  $N$  is the number of states  $\vec{Y}_i, r$  is a pre-assigned distance,  $\|\cdot\|$  is norm and  $\theta(y)$  is the Heaviside unit step function.

$$\theta(y) = \begin{cases} 0 & y < 0 \\ 1 & y > 0 \end{cases}$$

The sum in equation (2) gives the number of point pairs that are differed by a distance shorter than  $r$ . Grassberger-Procaccia’s main point is that  $C(r)$  goes as a power of  $r$  for small  $r$  i.e.,  $C(r) \propto r^v$ , where the exponent  $v$  denotes the correlation dimension which is related to fractal dimension  $D$ .

The correlation dimension in chaos theory is a measure of the dimensionality of space occupied by set of random points. For a set of random points on the real number line between 0 and 1, the correlation dimension will be  $v = 1$ . Instead, if a set of random points are distributed in a triangle embedded in three dimensional space, the correlation dimension will be  $v = 2$ . As stated earlier, the dimension of fractal objects can be determined from the correlation dimension. A time series of delay vector  $V$  in  $m$ -dimensional embedding space is constructed from the time series  $Y(i)$  as follows

$$V_m(i) = [Y(i), Y(i + \tau), Y(i + 2\tau), Y(i + \tau3), \dots \dots Y(i + (m - 1)\tau)]$$

When the number of points are sufficiently large and evenly distributed, then the ratio between the log of correlation integral and log of distance gives the correlation dimension  $v$ , which is often taken as equal to fractal dimension.

$$D_m = \frac{d \log C_m(r)}{d \log r} \tag{6}$$

where  $C_m(r)$  is the number of pairs of vector points,  $V_m(i)$  and  $V_m(j)$ , of which the mutual distance in the  $m$ -dimensional phase space is less than delay time  $\tau$ . GP algorithm is used to find the correlation dimension for the input data which are in a time series as given in equation (4).

## Results and Discussion

The fractal dimension analysis is executed with GP algorithm for 984 events. From the time series data of CNX Nifty, a state vector of sufficiently high dimensionality is constructed. Since the dimension is not known prior to the analysis, the dimension is sought by trial and error. The dimension of the embedded space is taken as large as seven. Since data sample is finite, considering too large dimension may cause statistical inaccuracy. So, the dimension is restricted up to seven.

Following Takens, the vectors are constructed as per the equation (4) and the correlation dimension is then derived from the distances between the points in embedded space following equation(6). The quantities  $\log C(r)$  and  $D_m$  are then plotted against  $\log(r)$  as depicted in the Figure 2. For a certain range of  $\log(r)$ , the points of  $\log C(r)$  versus  $\log(r)$  are observed to lie approximately on a straight line and the dimension is estimated as the slope of the straight line fitted to the data. Uncertainty of dimension may occur due to lack of sufficient data, noise etc. The results are shown in Figure 2 for CNX Nifty.

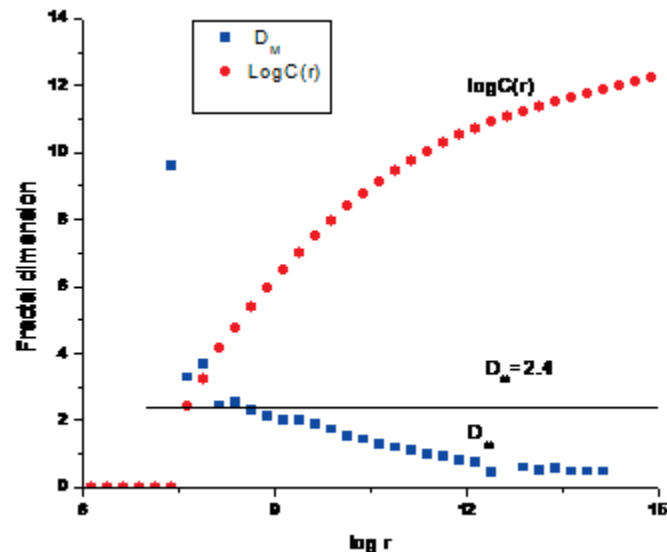


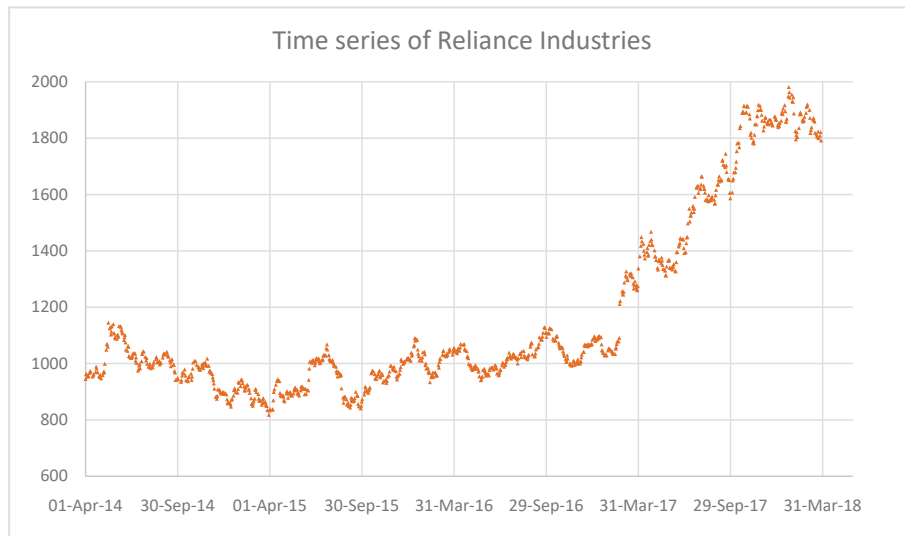
Figure 2: Variation of  $\log C(r)$  vs.  $\log r$  and  $D_m$  vs.  $\log r$  for 984 high price value of Nifty 50 over the period 01-04-2014 to 31-03-2018

It is noted that a few points of correlation dimension concentrate around 2.4 for CNX Nifty which indicate the system may contain strange attractor at the stated value in correlation dimension. The maximum Lyapunov exponent of these chaotic time series is simulated with Wolf's method getting the positive convergent value. This is another evidence for the chaotic feature of time series.

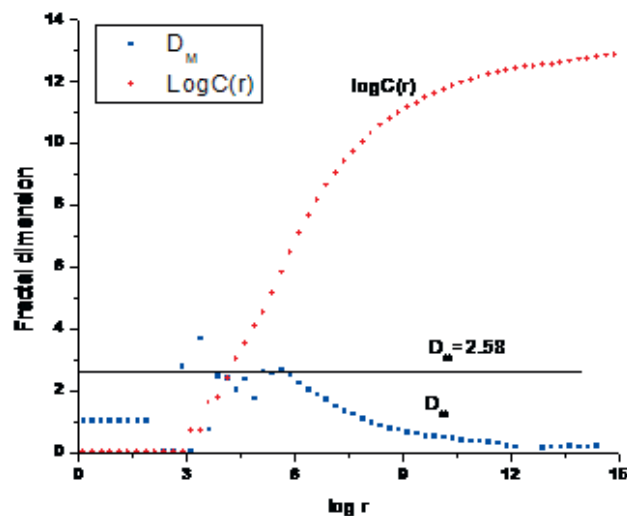
The analysis is repeated for equity of Reliance Industry which is currently the largest company of the country in terms of market capitalisation. The time variation of share price of Reliance Industry, which is shown in Figure 3, exhibits some distinct features compare to Nifty. The Reliance Industries Ltd. issued 1:1 bonus share on September 7, 2017. Consequently, share price of Reliance Industries became nearly half on September 7, 2017. Therefore, doubled market value of the share price of Reliance Industries from September 7, 2017 is taken to make the share price time series consistent over the time period considered in the analysis. Till March 2017 the share

price of Reliance Industry did not show much variation, it was moving around Rs. 1000/-. After that it has started to rise and in the last year the value of the stock has increased by roughly 100%. No significant effect of demonetization has been found in the stock price of Reliance Industry. It appears that GST has a positive effect on its stock price.

Following the same approach as done for Nifty, the correlation integral  $C(r)$  is evaluated and  $\log(C(r))$  and its derivative as a function of  $\log(r)$  are plotted in Figure 4. It is noticed that a few points of derivative of correlation integral concentrate around 2.58 which indicate the presence of fractal with dimension 2.58.



**Figure: 3** Time variation of daily high price of Reliance Industries Ltd. equity during 1st April 2014 to 31st March 2018. Note that to negate the bonus issue effect, the market value of share price of Reliance Industries is doubled from September 7, 2017



**Figure 4:** Figure 2 but for share price of Reliance Industries

It is noted that a few points of correlation dimension concentrate around 2.4 for CNX Nifty and around 2.58 for Reliance Industry which indicate the systems contain strange attractor at the stated values in correlation dimension.

## Discussion & Conclusion

In this work, the GP algorithm is used to examine whether any chaotic feature occurred in the Indian stock market over the last four years that witness a few unprecedented big economic events in the history of the country since its independence. The present findings suggest that both time series data of CNX Nifty index and share of Reliance Industries over the period April 2014 to March 2018 contain strange attractor. Here, it is noteworthy to mention that though price of global crude oil also suffered see-saw variation over last few years, no strange attractor was found in the crude oil price as reported by Sarkar, Paul & Majumder, (2015).

The time variation of CNX Nifty or stock price of Reliance Industries does not show any large unpredictable movements over the last four years. A few sectors such as real estate, consumer durables, PSU banking were hit hard by the demonetization but pharmaceuticals, IT, private banks reacted positively on demonetization. As a whole, the CNX Nifty was corrected by nearly seven per cent after demonetization but recovered soon and thereafter the market is increasingly going higher and higher. No significant immediate effect of the implementation of GST has been noticed in the movement of CNX Nifty. It is noticed that the share of Reliance Industries has been largely unaffected by demonetization, whereas stock seems to have reacted positively with the implementation of GST.

Under this context, the presence of strange attractor in the time series of CNX Nifty and stock price of Reliance Industries is significant. Does it suggest larger movement in the coming future? Small changes in the initial conditions lead to drastic changes in the output of a non-linear system but it requires sufficient time. The time period of four years is not large enough to reveal the implication of the underlying strange attractor. The current positive trend of Indian stock market is largely influenced by the global cues. It will be interesting to see the movement of Indian stock market when some saturation will come in the global market.

The dynamic behaviour of stock market is a very complex phenomenon, involving a large number of parameters including sentiments in global markets. In addition to demonetization and GST, several new factors such as, movement of Rupee against the Dollar, crude oil prices, etc. are influencing the sentiments of the share market. Introduction of so many factors may also be the reason for development of chaotic behaviour as in the case of weather. Consequently, the prediction of stock market dynamics and particularly accessing the risk factor becomes very difficult (Sarkar & Majumder, 2016).

The present analysis is restricted only to monofractal analysis. Recent analysis of financial markets suggests that monofractal description is not always sufficient to access its underlying multi-dimensional complexities (Jiang, Xie, Zhou & Sornette, 2018). In future, multifractal analysis will be employed for better access of the situation.

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### Author's Profile

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