



## **Use of Resource Performance Data to Tackle Uncertainty in Stochastic Nature of Activity Duration in PERT**

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### **Abstract**

Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM) are the two most basic techniques for project planning. The time variable in PERT is stochastic in nature. It takes three values i.e. the optimistic, pessimistic and most likely, then the expected value is calculated using a probability distribution whose graph matches the graph plotted by these three durations. The distribution is an approximation to the stochastic time variables representing the activity durations. This leads to a large research towards the prediction of probability distributions representing the activity durations with a greater accuracy. The concept of PERT was related to beta distribution originally. Researches have suggested various distributions and various modifications in existing distributions for a closer approximation of activity durations. Research to compare the effects of various distributions on the mean of activity duration is available which reveals a marginal difference in the prediction of expected activity duration, on varying the probability distribution. This implies that at the very first step of allocating time duration to the activities, uncertainty crops in two ways: one while predicting the three time estimates and other, while adopting the distribution with shape closest to the graph of these activity durations. Since the distributions are just approximations to the graph of activity durations, there is a need to find another method to fight uncertainty in activity durations. It is proposed to use the history of the performance of resources in similar projects to predict the pessimistic activity duration and buffer size. For every project, the project managers themselves decide the activity durations, still projects get delayed. This paper proposes to use the probability of delay in the project to recalculate the pessimistic duration and hence the estimated project duration. The same probability is also proposed to be applied to calculate buffer size. The probability of delay would be based on percentage delay in similar activities of older projects accomplished by resources being used for the proposed project.

**Keywords:** Program Evaluation and Review Technique (PERT), Critical Path Method (CPM), Activity Duration, Probability Distribution Curve

**JEL Classification:** C6

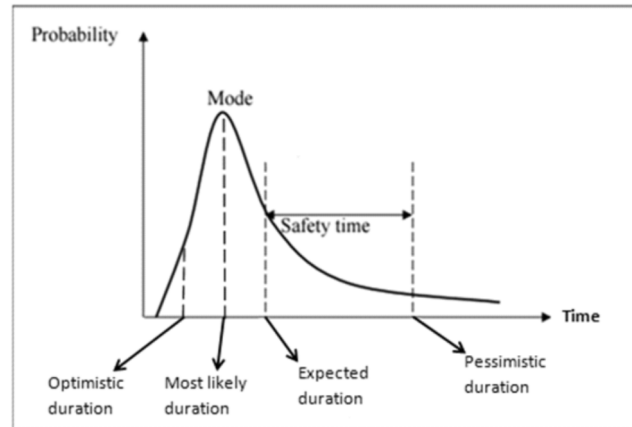
**Paper Classification:** Conceptual paper

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## Introduction

Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM) are the two most basic as well as significant tools for project management. PERT was developed in 1958 by the US Navy Special project officers in collaboration with Lockheed Aircraft Corporation and management consulting firms of Booz, Allen and Hamilton to complete the production of Polaris Missile in a record time. CPM was invented as a joint effort by the EI duPont Company and Remington Rand Univac in the period Dec 1956 to Feb 1959. Since the advent of these two techniques, they have been the base for scheduling the projects, though various changes have been suggested in the basic formulation of these techniques.

PERT considers the activity durations to be stochastic and hence estimates the activity durations using three estimates namely the optimistic, pessimistic and most likely activity duration i.e. instead of estimating fixed activity duration for each activity, a range  $[a,b]$  is assigned. The lower and upper bounds being the optimistic and pessimistic durations respectively and the most likely duration 'm' such that  $a < m < b$ . The three time durations for each activity are estimated by the project manager on the basis of past experience. This estimation is also affected by nature of the manager. The one courageous enough to take risks would allocate smaller durations to activities and hence the result would be smaller project duration but the one more cautious would add maximum possible time as security. The past experience of the manager is also a matter of chance. If the last project of the manager had gone well in time, it might instigate him to set smaller durations for each activity but the one who has experienced a significant delay in the completion of his past project would try to add maximum possible safety time as a precaution. The project manager estimates the pessimistic, optimistic and most likely durations for each activity. Optimistic duration relates to the situation when all goes fine and the activity finishes in minimum possible time. In contrast to this, the pessimistic activity duration concerns the situation when 'all goes wrong' (Murphy's law) and the activity finishes in maximum possible time. The third time estimate is the most likely time which is the realistic or the most probable activity duration. The expected duration for each activity is then calculated using an appropriate distribution. Mean of these three durations (pessimistic, most likely and optimistic) is taken as the activity duration, which is calculated using a probability distribution which the three estimates of activity duration are assumed to follow. Assumption regarding this probability distribution is made on the basis of shape which the graph of these estimates takes. In general the shape of graph is as shown in Fig 1. The position of expected duration in the curve may vary as per the values of optimistic, pessimistic and most likely durations. The graph shows a long tail of pessimistic duration with a small probability. This long tail makes the expected duration take a larger value, with a smaller probability. But it is seen that the activities still get delayed. To overcome this delay, it is proposed to further modify the pessimistic duration and it would further shift the activity duration towards right side. If the past history of similar activities indicate delay in completion (or left side if the activities in records finished before time). This might increase the activity and the project duration, but would make the project more robust.



**Fig. 1 Activity Duration Curve**

Researches have shown that on varying the probability distributions, the change in mean of activity duration is less than the change inculcated when activity durations are changed by 10% keeping the same distribution (Miklos Hajdu, 2014). In yet another paper Miklos Hajdu (2016), confirmed the same result using Monte Carlo Simulation. Hence a need for using some other methodology for predicting the possible delays is sensed.

Once the basic schedule is ready, it is required to add buffers at various stages of the schedule. A buffer is a cushion provided at different stages of a project schedule as it absorbs the delays that occur during execution of the project up to a maximum possible extent. In the development of the project plan, each activity will be coupled with a lot of security time, which mostly might be wasted. Even if a task is ahead of schedule, the security time will not accumulate to the next activity. Therefore, the concept of the buffer addition is introduced in the critical chain. It involves reducing the estimated time for each activity, which is accumulated and added in the form of buffers. There exist three types of buffers:

- I. The Project Buffer (PB): Project Buffers are inserted to protect the project delivery date.
- II. The Feeding Buffer (FB): Feeding Buffers are inserted at points where a non critical chain merges into a critical chain. It protects the critical chain from delays in the non critical chain.
- III. The Resource Buffer (RB) : Resource Buffers are inserted at every point where work passes from one resource to another.

There exist a number of methods for sizing of the buffer which are discussed in detail in the literature review. It can be observed that all the methods of buffer sizing are based on either the three values estimated to predict the activity durations or the measures of central tendency of the probability distribution assumed to represent the activity duration curve. This paper proposes to use the past records of the delay made by the resource in the completion of similar activity in past projects to calculate the buffers sizes.

## Literature Review

### PERT (Program Evaluation and Review Technique)

Literature shows that earlier beta or normal distributions were used to predict the activity duration. But later a number of researchers focused on the fact that normal approximation of

the activity durations required modifications. Some researchers proposed modification in the formulae for calculation of expected time using three parameters only, while others claimed it unrealistic to give three time estimates and proposed formulae using only two parameters.

Program Evaluation and Review Technique (PERT) is the most widely used project management technique and finds application in a wide variety of fields such as construction, production, software industry etc. Until the advent of PERT and CPM, there were no formal methods for project scheduling. Each project manager followed his own scheme. Literature shows the use of bar charts in those times but they were not enough to tackle more complex relationships among the activities and projects with larger number of activities (Joseph J.Moder, 1995). In 1958, the US Navy got an assignment to complete the production of Polaris Missile in a record time. It lead to the invention of PERT by special project officers of the US navy in collaboration with Lockheed Aircraft Corporation and management consulting firms of Booz, Allen and Hamilton (Malcolm et al. 1959). Traditional techniques such as line of balance, Gantt charts, and other systems were eliminated, and PERT evolved as a means to deal with the varied time periods it takes to finish the critical activities of an overall project (Cottrell, 1999). Since the time duration required by each activity to finish is not determined, PERT follows the three activity durations estimated by the project manager. The inventors of PERT proposed to follow a three point activity duration estimation scheme (Cottrell, 1999). The project manager estimates the optimistic duration (a), pessimistic duration (b) and most likely duration (m), which respectively represent the minimum possible, the maximum possible and the most probable time duration to complete an activity. In statistical terms, a hypothetical distribution is assumed to predict the activity duration with 'a' and 'b' as the extreme ends and 'm' as the mode, and then mean and variance is calculated as per the distribution obtained using 'a', 'b, and 'm'. Though initially beta distribution was used in PERT to predict the expected duration (Malcolm et al. 1959), a large research has been done later to find a distribution which better fits for this estimation. The research done in this area is majorly divided in three segments which are discussed in detail. In the first segment, researchers have proposed various distributions for the calculation of mean and variance of activity durations once 'a', 'b', and 'm' has been estimated. The second segment mentions the extra effort required in estimation of the three parameters and hence proposes distributions which use only two parameters for estimation of mean and variance of activity durations. Third segment comprises of researchers which say that no distribution fits best to all the activities of the project and hence rather than finding the best fitting of distribution, some more areas should be explored to reduce the effect of uncertainties in project schedule.

As proposed by the inventors of PERT (Malcolm et al. 1959), the activity duration follows a beta ( $\alpha, \beta$ ) distribution given by:

$$f(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(y - a)^{\alpha-1}(y - b)^{\beta-1}}{(b - a)^{\alpha+\beta-1}}, \quad a < y < b, \quad \alpha, \beta > 0 \quad (1)$$

Mean and variance (commonly written as  $\mu$  and  $\sigma$  respectively) of activity durations are given by the relations

$$\mu = t_e = \frac{1}{3} \left[ 2m + \frac{1}{2}(a + b) \right] \quad (2)$$

$$\sigma^2(t_e) = \left[ \frac{1}{6}(b - a) \right]^2 \quad (3)$$

C.E. Clark (1962), F.E. Grubbs (1962), T.L. Heally (1961), M.W. Sasieni (1986) questioned the formulae for mean and variance as these formulae cannot be directly calculated from (1). Considering the lack of theoretical proofs, L.Yu Chuen-Tao (1974) proposed a new expression for mean as

$$\mu=(a+km+b)/(k+2)$$

with the value of parameter  $k$  as 4. Rafael Herreras Pleguezuelo et al (2003) maintains that the accepted formulae of mean and variance of PERT distribution using beta distribution with weight equal to four given to the most likely activity duration is very much appropriate. Rafael Herreras Pleguezuelo (2003) further showed his agreement to this and gave proof for his claim that not the approximate but exact value of  $k$  is 4. Presenting a set of three conditions i.e. activity durations assume beta distribution, the beta distribution is assumed to have standard deviation as  $1/6$  of the range and coefficient of kurtosis equal to three, he claims that following all the three conditions the value of  $k$  comes out to be exactly 4 and considering first and second or first and third conditions values different than 4 give families of beta distribution, intersection of which again gives the PERT beta distribution. In another segment researchers tried to modify or introduce new parameters rather than 'a', 'b', and 'm'. J Berny (1989) proposed to use the probability to exceed the mode rather than pessimistic time along with the optimistic and most likely activity duration. In 1988, Golenko-Ginzburg D proposed to use a fixed value of the most likely activity duration i.e.  $m$  as  $m=(2a+b)/3$  and proposed to estimate the mean activity duration using the three parameters. He claimed that this assumption would simplify the calculation of activity duration. However this estimation may reduce the robustness of the schedule. Donald L. Keefer et al (1993) presented an approximation to calculate the mean and variance of activity durations (considering three time estimates) and claimed the results to be better than the conventional PERT methods. JMH Velaco et al (2011) proposed a method for the calculation of variance while retaining the original PERT mean expression and considering that the most likely value of activity duration aligns with beta distribution's mode. A number of researchers have supported lognormal distribution rather than beta distribution to estimate the activity duration curve. Dan Trietsch et al (2012) validated the lognormal distribution for activity duration and proposed Parkinson's distribution to fight hidden earliness. S. Mohan et al (2007) observed that two estimates i.e. either pessimistic and most likely time estimate or, optimistic and most likely time estimate are enough to predict the time estimate. He used lognormal distribution considering the right skewed nature of the activity duration curve. Williams (1992) proposed to use triangular distribution and Johnson (2002) further explored and validated it.

Premchandra (1999) proposed a procedure for estimating mean and variance in such a manner that no restrictions are imposed on the beta parameters 'a' and 'b'. He claims that the PERT assumptions are not violated in his procedure. Cottrell W (1999) proposed a simplified version of PERT using normal distribution and considering only the pessimistic and most likely activity durations. He claimed to have results similar to the conventional PERT methodology when the activity duration is not highly skewed, although the level of effort is reduced.

In another segment of the related research, Nasser Eddine Mouhoub (2011) proposed a new method to draw the PERT network with minimal number of arcs using the transitive arcs and some results on line graph. He claims that his method produces small project durations as project duration is proportional to the number of arcs. He proposes to reduce the number of dummy arcs as much as possible. Javier Castro et al (2008) proposed to collect all the slack on non critical chains and allocate it to the activities downstream the chain. Javier Castro et al (2007) defined a TU project game that satisfies some desirable properties from the management project and game

theory to allocate the total expedition (or delay) for situations in which the project is not executed as planned.

Miklos Hajdu et al (2014) investigated the effect of applying different distributions i.e. beta, triangular and uniform distribution to the same three point estimation of the activity durations and concluded that varying the distribution has lesser effect on the durations as compared to varying the estimates by 10%. They further confirmed this using lognormal distribution (Miklos Hajdu et al, 2016). Based on two observations i.e. the stochastically dependent processing durations can be linearly modelled using lognormal distribution and the historical data can be used to estimate the parameters for durations, Dan Trietsch et al (2012) presented a scheduling framework which they named as "PERT 21: PERT/CPM" which they claim to be more suitable for the modern decision support system. They propose it to be used parallel to the existing modules for risk management and suggest that further research is required for the development of their methodology.

## Buffer Management

Various methods have been proposed to manage buffer sizing. The C&P i.e. Cut and Paste Method and RSE i.e. Root Square Error Method are the most traditional ones. In Cut and Paste Method, half of the activity duration is cut from each activity on pretext of safety time added unnecessarily. These durations are then added and half the sum makes the buffer, hence adding 25% of the chain length as buffer. Root Mean Square Method uses two estimates for each task on the feeding chain; the safe estimate and the average estimate (Oya I tukel, 2006). Then the uncertainty of each task duration is calculated as  $U_i = S_i - d_i$  where  $U_i$  is the uncertainty of task  $i$ ,  $S_i$  is the safe estimate of task  $i$  and  $d_i$  is the average (50%) estimate of task  $i$ , for all  $i$  in the feeding chain. Newbold then suggests that the standard deviation in the task duration is  $(U_i/2)$  (Oya I tukel, 2006). Then the standard deviation of the feeding chain is:  $\sqrt{(U_1/2)^2 + (U_2/2)^2 + \dots + (U_n/2)^2}$  where  $n$  is the number of activities in the feeding chain. The assumption here is that the task completion time is independent. The buffer size is then two standard deviations:  $\text{Buffer size} = 2\sigma = \sqrt{(U_1)^2 + (U_2)^2 + \dots + (U_n)^2}$ .

Xue-mei Xai et al (2010) suggested Improved Root Square Error (IRSE), a method based on Critical Chain Theory for buffer sizing majorly suitable for software projects and the results claim to have a direct effect in shortening the project duration. Oya I. et al (2006) suggested two methods for determining feeding buffers; one incorporates resource tightness (APRT) while the other uses network complexity resulting in smaller buffer sizes. Adaptive Procedure with Resource Tightness (APRT) is used in cases of high resource crunch. Let  $r(i,q)$  be the resource required for activity  $i$  for resource type  $q$ ;  $D_i$  be the activity duration and  $Rav(q)$  be the amount of type  $q$  resource available;  $Var_i$  be the variance of activity  $i$  and  $T$  be the length of critical chain, then for each chain, size of the buffer is given by

$$\text{Buffer size} = k * \sqrt{\sum_i Var_i}$$

Where  $k = 1 + \max \left\{ \left( \sum_i r(i,q) * d_i \right) \frac{d_i}{T} * Rav(q) \right\}$  for each resource type  $q$ .

APD (Adaptive Procedure with Density) might be used as per requirement of the project. The APD procedure is used in cases where the network complexity is high while APRT is used in cases of high resource crunch.

The buffer size using APD is given by

$$\text{buffer size} = K * K * \sqrt{\sum_i \text{Var}_i} \quad \text{where } = 1 + (\text{TOTPRE} / \text{NUMTASK});$$

TOTPRE refers to total number of precedence relationships defined on the sub network feeding into critical chain and NUMTASK is the total number of tasks on the sub network and Var be the variance.

Mohammad Fallah (2010) proposed a buffer sizing technique using lognormal distribution for activities execution time. They considered inherent uncertainty of activities as a main factor to size the buffer. The algorithm they proposed is as follows

1. Calculate the following three parameters for each activity

$$\bullet \quad \frac{\sqrt{\text{Var} \bar{X}}}{E(X)} D = d_i (e^{\sigma^2} - 1) \left( \frac{\sigma^2}{2} \right) \quad (1)$$

$$\bullet \quad \text{Skewness} \times D = (e^{\sigma^2} + 2) \left( \frac{\sigma^2}{2} \right) \quad (2)$$

$$\bullet \quad \text{Kurtosis} \times D = (e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 6) * \left( \frac{\sigma^2}{2} \right) \quad (3)$$

Formulae have been simplified using the assumption given by Oya Tuckel

2. Arrange the obtained numbers in a descending order so that  $a(1) > a(2) > a(3)$

3. Calculate the numerical amount of the following convex combination:

$$\frac{a(1) - a(2)}{a(1) - a(3)} \times a(1) + \frac{a(2) - a(3)}{a(1) - a(3)} \times a(2)$$

Fuzzy numbers have also been used in methods to determine the project buffers. It has been observed that through Critical Chain Project Management, projects are completed in significantly shorter time than traditional Critical Path project management techniques. Importantly, Critical Chain Project Management is also simpler to use and requires less work for the project team in both the planning and tracking phases of project. M. Bevilacqua et al (2009) applied the Theory of Constraints and Risk Assessment to the turnaround process of an IGCC (Integrated Gasification Combined Cycle) plant. They developed a prioritization method for Work Packages, using the critical chain concept and concluded that the proposed method allowed the company to maximize the quality and safety of work and minimized the turnaround time and cost.

Junguang Zhang et al (2016) proposed a buffer sizing technique that takes into consideration the information resource tightness along with the physical resource tightness. It claims that his proposed method is more reasonable as it can doubly optimise the project duration and cost. Dorota Kuchta (2014) proposed a model using fuzzy numbers which aims at maintaining the project robustness and at the same time does not prolong the project duration too much. He suggests that only project managers should know of the buffers and not the ones actually executing the activities. This would reduce the effect of student syndrome. Vibha et al (2014) proposed a buffer placing technique to place the project and feeding buffers.

After calculating the buffer of each activity, the size of project buffer is obtained by adding

up the buffer of critical activities. The size of feeding buffer is determined through adding up the buffer of activities of each of the un-critical chains.

## Proposed Methodology

A survey of the present literature reveals that attempts have been made to find the best fit probability distribution for the activity duration. However, each probability distribution is an approximation and chances of deviation of the project from the proposed schedule cannot be ruled out. This suggests that rather than making efforts towards estimating the best fit probability distribution for activity duration, more fruitful would be to estimate the best values for parameters used to calculate the expected activity duration. This paper proposes to use the records of performance of the resources in similar kind of projects accomplished earlier. Dan Trietsch et al (2012) have mentioned the usage of records of delay in earlier projects to estimate the parameters for the calculation of activity duration using lognormal distribution. However, this research makes an attempt to address the root cause of delay in projects. Here an attempt is made to find out the performance of each resource in older projects and exploit its capability to the maximum in new projects. Furthermore it is proposed to estimate the buffer size also with the help of data recorded from the earlier projects. The data involves delay in each activity along with the resource which performed the activity. A detailed explanation of the data required to be recorded is elaborated in the points that follow. Any project when completed needs to be analysed to record the delays made by activities and the circumstances which lead to the delay.

1. Record the activities performed by each resource in various projects in terms of pairs of resources and activity types.
2. Record the percentage delay in the completion of each activity.
3. If the project runs under normal circumstances and still the activity got delayed, a weight equal to one would be assigned and if the delay took place because of certain unforeseen reason, repetition of which has a small probability, a smaller weight say 0.5 needs to be assigned.
4. The weight would be lesser for older projects and higher for newer ones.
5. The weights would be decided before the starting of any project plan as per the situations of the project to be planned.

An elaboration of the records to be maintained is done using hypothetical data. Table 1 shows the data required to be recorded. Let the project be divided into ten activities A-J. The past projects with similar activities would be chosen. Let the three resources R1, R2 and R3 be capable of performing activity A. List the performance of these resources in previous projects and calculate the weighted mean. Values under column R1 under activity A list the performance of resource R1 in 10 projects with similar kind of activity while the values under column R2 or R3 under activity A list the performance of resource R2 or R3 in some other sets of projects with similar kind of activity. A combination of resources is mentioned for activities which require more than one resource e.g. R6,R1 or R6,R2 under column D. This combination may be a team of human

resources or a combination of human resource(s) and machine(s). Similarly list the data for other activities. The resources would be allocated to activities based on their performances in previous projects in terms of decreasing order of performance.

**Table 1 : A Sample of Record of Resources' Performance in Past Projects**

Activity→	A			B			C		D		E		F	G			H		I		J
Resources →	R1	R2	R3	R1	R2	R3	R4	R5	R6,R1 R6,R2	R4	R5	R3	R1	R2	R3	R6,R1 R6,R2	R4	R5	R3		
Project (S.No.)↓	Percentage Delay																				
1	54	57	46	73	35	46	56	24	84	4	73	17	4	34	38	47	39	63	46	92	84
2	69	19	41	56	29	94	45	17	14	7	48	42	5	76	2	30	36	68	35	86	80
3	96	38	76	23	95	98	4	89	33	19	96	95	76	46	26	9	1	58	53	25	25
4	99	43	13	46	8	77	2	84	62	74	69	68	96	17	32	26	27	84	58	5	12
5	65	99	43	82	56	88	4	47	40	33	59	33	26	75	93	41	79	98	22	98	97
6	56	82	65	40	51	93	94	79	14	26	31	67	91	77	7	8	50	34	76	83	84
7	23	11	37	11	37	71	77	55	67	45	60	43	14	37	41	23	3	13	57	89	46
8	88	30	49	42	22	61	51	54	71	87	17	58	90	45	26	13	42	63	43	15	80
9	1	45	85	62	1	20	66	20	59	73	61	86	99	96	32	59	17	97	88	21	85
10	95	30	95	75	58	66	5	95	91	24	95	36	24	36	14	12	57	4	53	67	21
weighted mean→	46.1	29.8	45	43.6	32	46	25.3	47.7	46.75	27.55	56.1	38.6	37	35.9	22.7	19.4	25.8	41.35	44.7	42	37.8
max	96	99	95	82	95	98	94	95	91	87	96	95	99	96	93	59	79	98	88	98	97

**Table 2 : A Sample of Weights Allocated**

Activity→	A			B			C		D		E		F	G			H		I		J
project↓	R1	R2	R3	R1	R2	R3	R4	R5	R6,R1 R6,R2	R4	R5	R3	R1	R2	R3	R6,R1 R6,R2	R4	R5	R3		
	weights																				
1	1	1	2	2	2	2	1	1	2	1	2	2	2	2	1	2	1	1	2	1	1
2	2	2	1	1	1	1	1	1	2	2	1	2	2	1	2	1	1	1	2	2	1
3	2	1	2	2	2	2	1	2	1	1	2	1	1	2	1	2	1	1	1	1	2
4	1	1	2	2	1	1	2	2	1	2	2	1	1	2	1	1	2	2	1	1	2
5	1	1	2	2	1	1	1	2	1	2	2	1	1	1	2	2	2	2	2	1	1
6	1	2	1	1	1	1	2	1	2	1	1	2	2	1	2	1	1	1	2	2	2
7	2	2	2	1	2	1	1	2	2	2	2	2	1	2	1	2	1	1	1	2	1
8	2	1	2	1	2	2	1	1	2	1	1	2	2	2	1	2	2	2	2	1	1
9	1	1	2	2	1	1	1	2	2	1	2	1	1	1	1	2	1	2	1	1	1
10	1	2	1	2	2	1	2	2	2	1	2	1	2	1	2	1	1	1	2	1	2

Once the data has been recorded, it can be used to make the project more robust and allocate the activities to the resources optimally. The activity duration is proposed to be estimated according to the following rules:

- i. INITIALISE activity number=1
- FOR activity number=1 to number of activities

- ii. INITIALISE resource number=1  
FOR resource number=1 to number of resources  
IF delay in past record of this resource for similar activities is available
- iii. CALCULATE mean delay in the past projects
- iv. ELSE
- v. INCREMENT resource number by 1
- vi. GOTO III  
ENDIF  
ENDFOR
- vii. RECORD delay=min (mean delay)
- viii. RECORD the pair of activity and resource with minimum delay as resource allocated for corresponding activity.
- ix. READ a=optimistic, m=most likely, b= pessimistic duration, d=percentage average delay
- x. COMPUTE  $b_1=(1+d)b+d(a+4m)$
- xi. COMPUTE expected activity duration using any probability distribution using the modified pessimistic duration, the optimistic and most likely durations
- xii. INCREMENT activity number by 1
- xiii. GOTO II  
ENDFOR

There exist various formulae for buffer sizing in literature as discussed in literature review. It can be observed that almost all the formulae of buffer sizes are based on statistical measures of the three activity durations. But in this paper, a different criterion for estimating the buffer size is suggested as the variance has already been used while estimating the expected duration. It is proposed to use the delay records of previously completed projects to calculate the buffer size using the formula

$$\sum_{i=1}^n \frac{D_i}{100} \times t_i$$

Where  $D_i$  is the percentage mean delay of activity  $i$ ,  $t_i$  is the activity duration and  $n$  is the number of activities on the chain for which buffer is to be calculated. The chain to be identified depends on the type of buffer to be added. There are three kinds of buffers i.e. the project buffer, feeding buffer and the resource buffer. A project buffer is inserted to protect the project delivery date. The feeding buffers are inserted at points where a non critical chain merges into a critical chain. It protects the critical chain from delays in the non critical chain. The resource buffers are inserted at every point where work passes from one resource to another.

However if the records of delay in previous projects' activities are not available, any buffer sizing techniques can be used.

As the pessimistic duration, is modified, each activity duration gets increased and hence the project duration too. But considering the concern of meeting deadlines in projects is more important, the increased activity duration is justified. Another point to be proposed is that the resources be given the activity durations without modifying the pessimistic duration and

the modified duration be known to the project manager alone. The deadline of the project to be announced should be according to the modified durations. This will automatically insert a buffer of size

$$\sum_{i=1}^n d_i$$

in the project. Allocating the resources unmodified duration would also avoid the delays due to student's syndrome to occur.

## Conclusion

The purpose of scheduling is to predict the completion time of project. Even after considering the elements of uncertainty in scheduling while estimating the activity durations, the projects tend to delay. Hence rectification of the estimates made by the project manager makes sense. Though the proposed methodology increases the project duration marginally, but the advantage of robustness compensates the increase. Moreover the optimal allocation of the resource according to the proposed methodology contributes to the optimal usage of resources. Sizing the buffer in terms of delay made by same resource in previous projects would be the optimal since it would be resource specific rather than based on some general statistical theory. The size of buffers is calculated using the previous record of resources. This makes sense because the variance in estimating the activity duration has already been considered while calculating the mean activity duration. Although the uncertainty is considered while estimating activity duration, activities still get delayed. Moreover, while considering the past data, delay in individual activities is considered and if other activities have an effect on its performance, a lesser weight given to that activity confirms that only resource's performance in that activity is monitored but not the project delay. A summary of the project finished is always maintained in excel sheet and hence maintaining performance of resources will not be a major task. Hence this methodology is practical and would produce better results. As per the proposed methodology, buffer for each activity would be assigned as per its own characteristics and ability of the resource allocated to it handles that very activity. Though the buffer would be accumulated at the end of the chain, each activity will have its appropriate share in the buffer.

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